Practical applied mathematics

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PRACTICAL
APPLIED MATHEMATICS

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BY

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PREFACE

This book together with that on "Practical Mechanics and Allied Subjects" by the same author embodies the results of five years' experience in the organization and development of a system of railroad shop apprentice schools, as well as extended investigations of the work of public, private, trade, vocational and corporation schools of manufacturing industries and railroads in this country and abroad.

In the presentation no formal distinction is made of the various branches of elementary mathematics and science involved. The subject matter is presented in the two volumes under the general titles of "Practical Applied Mathematics" and "Practical Mechanics and Allied Subjects," although a careful and logical arrangement and sequence have been followed in the presentation.

In organizing trade apprentice schools difficulty has been experienced in securing a text practically adapted to the needs of these schools. This difficulty has led the author to develop a series of instruction sheets which are believed to be thoroughly practical. This volume with that on "Practical Mechanics and Allied Subjects" is essentially a presentation of problems arising in shop experience in the mechanical trades. It shows the use of mathematics as a tool.

The material included in this work has been made sufficiently broad to apply not only to railroad schools but it is believed to schools in the mechanical trades generally. It is also felt that this work may be used to advantage in the public schools in technical branches, as well as in trade and vocational schools, either as a regular or as a supplementary text.

The material for each book is presented in twenty chapters, each dealing with a definite subject. This affords a flexibility highly desirable in assigning the work and enables it to be given entirely or in part according to specific needs.

In preparing this work the author acknowledges indebtedness to Pennsylvania Railroad Motive Power Officials at Altoona. The author is indebted to Dr. J. P. Jackson, State Commissioner of Labor and Industry of Pennsylvania, Professor J. A. Moyer, Head of the Department of Mechanical Engineering and Director of the Engineering Extension Division of the Pennsylvania State College and Mr. M. B. King, State Expert Assistant in Industrial Education of Pennsylvania, for reading the proofs and offering valuable criticisms and suggestions. He is especially indebted to Professor J. H. Yoder and Mr. E. W. Hughes of the School of Engineering of the Pennsylvania State College, detailed as Instructors in the Pennsylvania Railroad System of Apprentice Schools. They have assisted in the preparation of the manuscript, carefully read the proofs and offered many valuable suggestions.

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PRACTICAL APPLIED MATHEMATICS

CHAPTER I

SIMPLE PROCESSES

1. Addition.—It is often necessary in the work of the shop to find the sum of two or more numbers or quantities. For example, if 3 ft. of bar metal are required for one job and 4 ft. of the same metal for another job, the two jobs together require 3 + 4 or 7 ft. of bar metal. This shows how much material is required all together and from this the order can be properly placed. This process of finding the sum of two or more numbers or quantities is called "addition."

In addition it must be remembered that only similar kinds of quantities can be added together to get a correct result. For example, we cannot add pounds and yards and get a single number representing their sum any more than we can add locomotives and rails and get a single number representing this sum.

For example, if we add 2 yd. and 3 ft., to 4 yd. and 8 in., we must put them down as follows:

	2 yd. 3 ft. 0 in.
	4 yd. 0 ft. 8 in.
Sum	6 yd. 3 ft. 8 in., or 7 yd. 8 in.

In other words, we must keep like units under like units in our columns of addition.

In a number the places have the names as given below:

Billions	Hundred Millions	Ten Millions	Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Units
8,	4	3	7,	9	6	4,	8	7	2

This number is therefore read eight billion, four hundred thirty-seven million, nine hundred sixty-four thousand, eight hundred and seventy-two.

In adding numbers, place them in a vertical column putting units under units, tens under tens, and so forth. For example, if we have five iron castings weighing as follows: 8 lb., 42 lb., 163 lb., 4327 lb., 16 lb., and wish to find their combined weight we arrange the numbers as follows:

$$\begin{array}{r} 8 \\ 42 \\ 163 \\ 4327 \\ 16 \\ \hline 4556 \end{array}$$

The first column adds up to 26 which equals two tens and six. We therefore put the 6 in the units column and add the two tens to the tens column and so on getting a sum of 4556 lb. which is the weight of all the castings taken together.

PROBLEMS

1. Find the sum of the following: 12 lb., 87 lb., 9320 lb., 643 lb.

2. After being machined a casting weighs 210 lb. The metal turned off weighs 34 lb. What was the weight of the rough casting?

3. The following distances have to be laid out in a straight line on a drawing: 4 in., 2 in., 11 in., 17 in. What is the overall dimension?

4. Find the total weight of a train made up as follows:

Engine and tender	350,000 lb.
Baggage car	80,000 lb.
1st coach	106,400 lb.
2nd coach	108,200 lb.
3rd coach	109,800 lb.
Parlor car	112,000 lb.
Sleeping car	114,000 lb.

5. Four steel bars the lengths of which are 29 ft., 34 ft., 19 ft., and 9 ft., are placed end to end. How far do they reach?

6. Add
```
  96
 432
 795
8643
   4
----
```

7. Add
```
   39
  627
   81
43728
   65
   18
-----
```

8. Add eighty-nine, two thousand four hundred sixty, seventeen, and forty-four.

2. Subtraction.—The process of finding the difference between two numbers is called "subtraction," which is the reverse of addition.

Thus: If a casting weighs 230 lb. after machining, how much metal was machined off if it weighed 246 lb. in the rough?

In this case the metal machined off would be the difference between 246 lb. and 230 lb. or 16 lb.

This may be written 246 lb. − 230 lb. = 16 lb.; the sign (−) read "minus" means that 230 is to be taken from 246.

If the numbers are put down as follows:

246	Minuend
230	Subtrahend
16	Difference

The upper number, 246, is called the minuend, the lower number, 230, the subtrahend, and the result of the subtraction is called the difference. As in addition, it is best to place the numbers to be subtracted *directly* under each other, putting units under units, tens under tens, etc.

Example.—If from a stock of 428 bolts 319 are used, how many are left?

We set the problem down as follows:

428	Minuend
319	Subtrahend
109	Difference

Beginning at the right we see that 9 cannot be taken from 8, hence we take 1 from the 2 and call the first number 18 instead of 8, leaving 1 in the tens place in the upper number. Now 9 from 18 leaves 9, 1 from 1 leaves 0 and 3 from 4 leaves 1. Hence the answer or difference is 109 bolts left.

PROBLEMS FOR PRACTICE

Subtract:

9. 384	**10.** 924	**11.** 976	**12.** 809
179	398	541	761

13. Subtract 789 from 9876.

14. Subtract 972 from 1896.

15. Find the value of 897 − 864.

16. Find the value of 3976 − 2094.

17. A locomotive weighing 270,000 lb. has 94,700 lb. on the leading and trailing trucks. How much weight is on the drivers?

18. From five thousand seven hundred eighty-two subtract two thousand seven hundred fifty-seven.

19. A rough casting weighs 109 lb. and after it is machined it weighs 87 lb. How much metal is removed in machining?

3. Multiplication.—A number may be used once, or any number of times. Thus 2 taken once is 2, and 2 taken 4 times is 2 + 2 + 2 + 2 or 8. Finding a quantity which is equal to some number taken a certain number of times is called "multiplication." Multiplication is simply a shortened method of adding equal numbers. The number to be multiplied is called the multiplicand. The number indicating how many times the multiplicand is taken is called the multiplier, and the result is called the product.

The following table gives all the numbers multiplied together (2 at a time) from 1 to 12 inclusive. The number in any block is equal to the number in the top row multiplied by the number of the row.

In this way small numbers may be multiplied by referring to this table, although everyone ought to know the products by memory for numbers as high as 12.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3		9	12	15	18	21	24	27	30	33	36
4			16	20	24	28	32	36	40	44	48
5				25	30	35	40	45	50	55	60
6					36	42	48	54	60	66	72
7						49	56	63	70	77	84
8							64	72	80	88	96
9								81	90	99	108
10									100	110	120
11										121	132
12											144

To multiply numbers with two or more figures we proceed as follows:

Example.—Multiply 2768 by 2034

```
   2768 Multiplicand
   2034 Multiplier
  -----
  11072
  8304
 5536
-------
5,630,112 Product
```

Multiply the multiplicand (2768) by 4, the first number in the multiplier, and place the result below the multiplier, placing units below units, tens below tens, etc. Next, multiply 2768 by 3, the second number in the multiplier, and place the result below the first product beginning, however, one space to the left or at the tens place. Since any number multiplied by zero is zero we need not put anything down for the product of 0 times 2768 but multiply at once by 2, placing our first number under the fourth column. The sum of these partial products will be the desired result.

The sign (×) read "times" or "multiplied by" is used to indicate multiplication.

PROBLEMS FOR PRACTICE

20. A casting weighs 643 lb. Find the weight of 274 such castings.

21. A train of 24 cars is loaded with car wheels each weighing 518 lb. What is the total weight of all the wheels if there are 97 wheels on each car?

22. Multiply 786 by 42
Multiply 384 by 75
Multiply 3786 by 5206
Multiply 38976 by 7284.

23. Find the value of $7 \times 9 \times 16$
Find the value of $8 \times 6 \times 13$
Find the value of $9 \times 27 \times 845$.

24. An order for pipe is as follows:

7 pieces	5 ft. long.
115 pieces	12 ft. long.
13 pieces	15 ft. long.
25 pieces	11 ft. long.
119 pieces	9 ft. long.

Find the total number of feet of pipe in the entire order.
Find weight of pipe at 9 lb. per ft. and the cost at 6 cts. per lb.

25. If 38 lb. draw-bar pull are required to haul 1 ton on a certain track, what would be the total draw-bar pull required to haul a train of 43 cars over this track if the average weight per car is 39 tons?

26. A machinist on piecework receives 17 cts. per piece. What is his income for 1 week (6 days) if he averages 21 pieces per day?

27. How many inches are there in 6 miles?

28. If 3520 ties are required to lay 1 mile of single track, what would be the cost of the ties at 95 cts. apiece to lay 356 miles of single track?

29. If 1 man can ballast 60 ft. of track in 1 day, how many feet of track can 12 men ballast in 3 weeks of 6 days each?

30. One man can assemble 2 machines per day. How many machines can 19 men assemble in 13 days?

31. How many feet of No. 10 wire are there on a reel containing 175 lb. if No. 10 wire measures 32 ft. per lb.?

32. A man receives 28 cts. per hr. How much would he earn in a year if he works 10 hr. per day, 6 days per week, and 50 weeks per year?

4. Division.—It often becomes necessary to find out how many times one number is contained in another. This process is called "division."

Division is the reverse of multiplication and is a shortened process of subtraction. If it is desired to find how many 3-in. pieces can be sheared from a bar 46 in. long we could subtract 3 a number of times until the number left was smaller than 3. By trial we find that 3 could be subtracted 15 times from 46 with a remainder of 1 in. of bar. Hence we could cut 15 pieces 3 in. long from the 46-in. bar and have 1 in. left over. By division this problem is worked out as follows:

```
3)46(15 pieces
  3
  --
  16
  15
  --
   1 remainder
```

Here we cannot tell at once how many times 3 is contained in 46 but we can see at once that 3 is contained in 4 once. We arrange our figures as shown and place the 1 to the right, multiply the 3 by 1 which gives 3 and place this under the 4. By subtracting we have 1 left. The next step is to bring down the 6 placing it beside the 1 making the new number 16. 3 in 16 goes 5 times. This 5 we place to the right of the 1 and multiply the 3 by 5 which gives 15 to be written below the 16, leaving a remainder of 1.

The 3 is called the divisor, the 46 the dividend and the 15 the quotient.

The sign (÷) read "divided by" indicates division.

PROBLEMS FOR PRACTICE

33. How many castings each weighing 117 lb. can be loaded on a car whose capacity is 60,000 lb.?

34. How many rails each 30 ft. long are required to lay 1 mile of single track?

35. Divide 24 by 2.
Divide 376 by 46.

36. Find the value of 728 ÷ 32.
Find the value of 685 ÷ 35.
Find the value of 8962 ÷ 46.

37. How many ties placed 18 in. between centers are required per mile of single track?

38. Into how many lots of 45 lb. each can 3756 lb. of castings be divided and how many pounds will be left over?

39. How many pieces 5 in. long can be cut from an iron rod 78 in. long? No allowance is made for waste in cutting.

40. A wrought-iron pipe 15 ft. long weighs 135 lb. What is its weight per ft.?

41. A job requiring 398 hr. is divided equally among 36 men. How many even hours must each man work and how much overtime would 1 man have to work to finish the job?

42. If 26,240 lb. of castings were loaded in a freight car whose capacity is 80,000 lb., how many castings each weighing 118 lb. can be added in order that the car may be fully loaded?

43. If a lathe is capable of finishing 4 shafts per day and an order is received for 160 shafts to be delivered in 5 days, how many machines must be used?

44. Cast iron weighs 450 lb. per cu. ft. How many cubic feet in a casting weighing 16,120 lb.?

45. How many rails each 30 ft. long will be required to lay 2 miles of double track? If rails weighing 100 lb. per yd. are used what will be the cost of the rails at $28 per ton?

46. If 1 man can ballast and dress 60 ft. of track in 1 day how many men will be required to ballast and dress 2 miles of track in 4 days?

47. If 34 lb. draw-bar pull is required to haul a ton on a certain track, how many 45-ton cars can be hauled with a locomotive whose draw-bar pull is 60,000 lb.?

48. A load of castings is weighed in six lots as follows: 276 lb., 397 lb., 486 lb., 298 lb., 875 lb., 962 lb. What is the total weight of the load?

49. If 1532 lb. of the castings in Prob. 48 are used immediately how many pounds are left?

50. An order of castings weighs as follows: 37 pieces 15 lb. each, 51 pieces 11 lb. each, 73 pieces 12 lb. each, 66 pieces 14 lb. each and 19 pieces 104 lb. each. What is the total weight of the lot?

51. Find the total number of hours work which has been done on light repairs on a locomotive and the cost of repairs at an average of 27 cts. per hr. as follows:

8 men 9 hr. each.
2 men 38 hr. each.
4 men 56 hr. each.
2 men 4 hr. each.
6 men 24 hr. each.

52. A job of piping requires the following lengths of pipe:

6 pieces 11 ft. long.
8 pieces 9 ft. long.
12 pieces 8 ft. long.
6 pieces 4 ft. long.
25 pieces 16 ft. long.
19 pieces 13 ft. long.

Find the total weight at 9 lb. per ft.

53. Rails weighing 85 lb. per yd., on a single track 16 miles long, are replaced by rails weighing 100 lb. per yd. How much more will these rails weigh for the 16 miles? (1 mile = 1760 yd.)

54. An order for structural steel calls for the following:

18–6-in. steel channels, 17 ft. long 8 lb. per ft.
24–9-in. steel I beams, 18 ft. long 21 lb. per ft.
25–4 in. × 3-in. angles, 16 ft. long 10 lb. per ft.

Find the total weight.

55. If 100 long tons (2240 lb.) of coal are bought at $4 per long ton and sold for $4 per short ton (2000 lb.), what is the profit made on the transaction?

56. Find the cost of the following bill of lumber at 3 cts. per board ft.:

125 joists	containing	34 board ft. each.
15 girders	containing	100 board ft. each.
62 rafters	containing	17 board ft. each.
13 posts	containing	35 board ft. each.
258 scantlings	containing	10 board ft. each.
300 boards	containing	11 board ft. each.

57. A rough forging for a shaft weighs 256 lb. After machining the shaft weighs 203 lb. If the forging originally cost 18 cts. a lb. but the metal machined off can be sold for 3 cts. a lb., what is the actual cost of the metal in the shaft?

58. Which would be the cheaper and how much, to employ a man at 28 cts. an hr. who takes 52 hr. to do a piece of work or to employ a man at 19 cts. an hr. who takes 76 hr. to do the same work?

59. If ties are placed 18 in. apart from center to center how far will 20,000 ties reach?

60. If it costs $50,000 on an average to do away with one grade crossing how much will it cost to eliminate 13,000 grade crossings?

61. How many rails, each 30 ft. long, are required for four tracks (8 rails) from Pittsburgh, Penna., to Chicago, Ill., a distance of 490 miles? (One mile contains 5280 ft.)

CHAPTER II

COMMON FRACTIONS

5. Definitions.—In any case where we measure a thing with a given unit like a foot, and we get a result which does not come out a whole number of units, we must use fractions. In other words, all measurements do

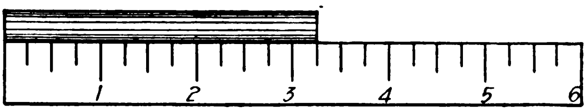


FIG. 1.

not come out in an even number of feet, or inches, yards or miles. For example, in measurements less than an inch we use "fractions" of an inch.

Figure 1 shows a 6-in. scale. The object being measured is 3 whole inches and $\frac{1}{4}$ in. in length. In the scale shown in Fig. 2 part A is divided into 4 equal parts. Each part is therefore $\frac{1}{4}$ in. (one-fourth of an inch). Each of the smallest lengths in part

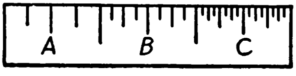


FIG. 2.

B is $\frac{1}{8}$ in. (one-eighth of an inch). Each of the smallest lengths of part C is $\frac{1}{16}$ in. (one-sixteenth of an inch).

These parts "$\frac{1}{4}$," "$\frac{1}{8}$," and "$\frac{1}{16}$" are called "fractions." These may also be written "$\frac{1}{4}$," "$\frac{1}{8}$," and "$\frac{1}{16}$." In the fraction "$\frac{1}{4}$," the lower number "4" called the "denominator" indicates into how many parts the unit is divided while the upper number or "numerator" indicates how many of these parts are taken. Three of these parts would be written "$\frac{3}{4}$." Fractions whose numerator is less than the denominator are called *proper fractions*. Fractions whose numerator is larger than the denominator are called "*improper fractions*," like $\frac{14}{8}$. Numbers made up of a whole number and a fraction like $4\frac{3}{4}$ are called "*mixed numbers*."

6. Reducing to Lower Terms.—From the scale shown above we can readily see that $\frac{1}{4} = \frac{4}{16}$, $\frac{3}{4} = \frac{12}{16}$, also $\frac{4}{16} = \frac{1}{4}$, and $\frac{12}{16} = \frac{3}{4}$. Hence we can multiply or divide both numerator and denominator of a fraction by the same number without changing the value of the fraction. Thus,

$$\frac{1 \times 4}{4 \times 4} = \frac{4}{16} \text{ and also } \frac{4 \div 4}{16 \div 4} = \frac{1}{4}$$

The last process is called reducing to lower terms.

7. Lowest Common Denominator. Addition, Subtraction.—What part of an inch would 1 part on A + 3 parts on B + 5 parts on C make?

This could be written $\frac{1}{4}$ in. + $\frac{3}{8}$ in. + $\frac{5}{16}$ in.

We see at once that we cannot add these the way they stand. We cannot add 4ths and 8ths directly to obtain a single fraction any more than we can add bolts and locomotives and get all bolts or all locomotives. *We must reduce them all to the same denominator.* We know that $\frac{1}{4} = \frac{4}{16}$ and $\frac{3}{8} = \frac{6}{16}$. Then $\frac{4}{16} + \frac{6}{16} + \frac{5}{16} = \frac{15}{16}$ in. Here 16 is called the *lowest common denominator.*

Fractions, however, do not always come in 4ths, 8ths, and 16ths. When we want to add fractions we must reduce them all to fractions having the same denominator.

For instance, if we want to know what part of a carload there is in three cars if the first is $\frac{1}{2}$ full, the second $\frac{1}{3}$ full, and the third $\frac{3}{4}$ full, we must add $\frac{1}{2} + \frac{1}{3} + \frac{3}{4}$ and our denominator must be a number that can be evenly divided by 2, 3, and 4. This number is 12. $\frac{1}{2} = \frac{1}{2} \times \frac{6}{6} = \frac{6}{12}$, $\frac{1}{3} = \frac{1}{3} \times \frac{4}{4} = \frac{4}{12}$, $\frac{3}{4} = \frac{3}{4} \times \frac{3}{3} = \frac{9}{12}$. Their sum equals $\frac{6}{12} + \frac{4}{12} + \frac{9}{12}$ or $\frac{19}{12}$ of a carload. Since one whole carload contains $\frac{12}{12}$, $\frac{19}{12}$ would make $1\frac{7}{12}$ carloads.

PROBLEMS FOR PRACTICE

1. Add $\frac{1}{2} + \frac{1}{16}$ $\qquad$ $\frac{1}{3} + \frac{1}{4}$

$\frac{3}{4} + \frac{5}{6}$ $\qquad$ $\frac{2}{3} + \frac{3}{4}$

$\frac{7}{8} + \frac{2}{3}$ $\qquad$ $\frac{2}{3} + \frac{1}{2} + \frac{1}{3}$.

2. An apprentice completes $\frac{1}{4}$ of a job and a machinist completes $\frac{1}{3}$ of it. What part of the whole job is completed? What part still remains to be done?

3. Find the overall length for the template in Fig. 3.

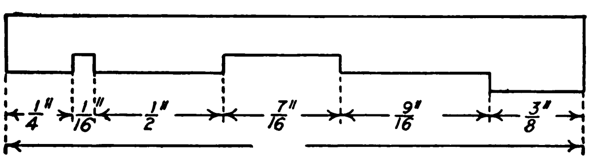


FIG. 3.

4. A keg of rivets $\frac{1}{4}$ full is emptied into another keg $\frac{2}{3}$ full, the kegs being of the same size. How full is the second keg?

5. Four castings weigh as follows: $\frac{1}{4}$ lb., $\frac{3}{8}$ lb., $\frac{5}{16}$ lb., and $\frac{1}{8}$ lb. What is their combined weight?

In many cases the common denominator cannot be found by mere inspection. For instance, if we wish to add $\frac{3}{16} + \frac{5}{18} + \frac{7}{12}$ we must find the lowest common denominator in the following manner:

$$\begin{array}{r|rrr} 2 & \frac{3}{16} + & \frac{5}{18} + & \frac{7}{12} \\ \hline 2 & 8 & 9 & 6 \\ \hline 3 & 4 & 9 & 3 \\ \hline & 4 & 3 & 1 \end{array} \quad = \frac{27}{144} + \frac{40}{144} + \frac{84}{144} = \frac{151}{144}$$

$$\frac{151}{144} = 1\frac{7}{144}$$

and the lowest common denominator (abbreviated L.C.D.) equals $2 \times 2 \times 3 \times 4 \times 3 = 144$.

Rule.—Find the smallest number other than 1 which will exactly divide two or more of the numbers whose L.C.D. is sought. Divide it into all the denominators that are exactly divisible by it. Write the quotients below and also all the numbers which are not exactly divisible by the divisor used. Repeat this operation until no number can be found which will exactly divide into any two of the numbers. The continued product of the divisors and the numbers remaining after the last division is the lowest common denominator.

MISCELLANEOUS PROBLEMS

6. Add $\frac{7}{8} + \frac{5}{24} + \frac{9}{32}$.
Add $\frac{8}{15} + \frac{7}{48} + \frac{5}{32}$.
Add $\frac{4}{15} + \frac{7}{8} + \frac{9}{10}$.

7. Subtract $\frac{3}{32} - \frac{1}{16}$.
Subtract $\frac{24}{35} - \frac{1}{7}$.
Subtract $\frac{4}{25} - \frac{7}{125}$.

8. Find the missing dimensions in Fig. 4, on the following page.

9. The center of a quarter inch hole is $\frac{23}{32}$ in. from one side of a brass strip $\frac{63}{64}$ in. wide. How far is the center of the hole from the other side of the strip?

10. A strip of steel plate $1\frac{5}{16}$ in. wide has a $\frac{5}{8}$-in. hole drilled in the center. How much metal is left on each side?

8. Multiplication of Fractions.—We know that $\frac{1}{8}$ in. $= \frac{2}{16}$ in., $\frac{1}{4}$ in. $= \frac{4}{16}$ in. and $\frac{1}{2}$ in. $= \frac{8}{16}$ in. Since it takes two sixteenths to make $\frac{1}{8}$ we see that $\frac{1}{16}$ is $\frac{1}{2}$ of $\frac{1}{8}$, or in other words $\frac{1}{2}$ of $\frac{1}{8}$ in. $= \frac{1}{16}$ in. "Of" means "multiplied by."

In like manner $\frac{1}{4}$ of $\frac{1}{4} = \frac{1}{16}$ and $\frac{1}{8}$ of $\frac{1}{2} = \frac{1}{16}$. If $\frac{1}{8}$ of $\frac{1}{2} = \frac{1}{16}$, then $\frac{2}{8}$ of $\frac{1}{2}$ would be twice $\frac{1}{16}$ or $\frac{2}{16}$ or $\frac{1}{8}$. Hence, if we wish to multiply two fractions together we write the product of the numbers above the line as the numerator and the product of the denominators below the line as the denominator of the product of the fractions.

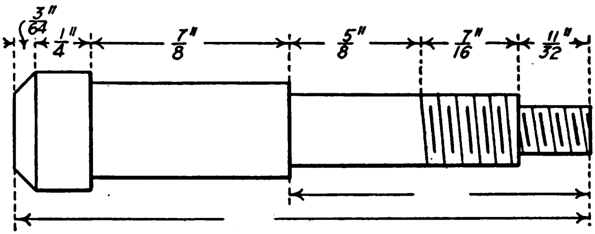


FIG. 4.

Thus: If from a car that is $\frac{3}{4}$ full of coal $\frac{2}{3}$ of this is taken out, what part of a carload is taken out?

$$\frac{2}{3} \text{ of } \frac{3}{4} = \frac{6}{12} \text{ or } \frac{1}{2} \text{ carload.}$$

Our work will often be shortened if we divide a number above the line into one below the line, or the other way around. Thus:

$$\frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{3}}} \text{ of } \frac{\overset{1}{\cancel{3}}}{\underset{2}{\cancel{4}}} = \frac{1}{2}$$

This is called "cancellation."

PROBLEMS FOR PRACTICE

11. Find the value of: $\frac{3}{4} \times \frac{7}{8}$; $\frac{5}{8}$ of $\frac{6}{25}$; $\frac{3}{7}$ of $\frac{14}{27}$; $\frac{5}{8}$ of $\frac{9}{14}$; $\frac{9}{11}$ of $\frac{22}{36}$.

12. A man receives \$$\frac{1}{3}$ an hr. How much will he get for $\frac{3}{4}$ hr.?

9. Division of Fractions.—In dividing fractions we invert the divisor and proceed as in multiplication. Thus:

$$\frac{3}{4} \div \frac{2}{3} = \frac{3}{4} \times \frac{3}{2} = \frac{9}{8} \text{ or } 1\tfrac{1}{8}$$

PROBLEMS

13. How many bolts, each weighing $\frac{1}{3}$ lb., are there in a lot weighing 30 lb.?

14. A man who gets $\frac{3}{8}$ of a dollar an hr. receives $\frac{7}{8}$ of a dollar for doing a piece of work. How long did it take him to complete the job?

15. Divide $\frac{9}{16}$ by $\frac{3}{4}$; $\frac{7}{8}$ by $\frac{3}{4}$; $\frac{9}{25}$ by $\frac{3}{20}$.

16. Find the value of $\frac{5}{3} \div \frac{7}{8}$; $\frac{3}{5} \div \frac{9}{20}$; $\frac{3}{16} \div \frac{9}{32}$.

10. Mixed Numbers.—A mixed number, composed of a whole number and a fraction as $2\frac{1}{2}$ in. $3\frac{1}{3}$ miles, $4\frac{1}{5}$ tons, etc., may be reduced to an improper fraction by multiplying the whole number by the denominator of the fraction, adding to the product the numerator of the fraction, and writing this sum above the denominator, thus:

$$4\tfrac{3}{5} = \frac{(4 \times 5) + 3}{5} = \frac{23}{5}$$

11. Tables of Units.—In working out the problems which follow we shall need to understand how measurements are made and the "units" that are used. For example, to measure length we use the inch, foot, yard, rod or mile. To measure time we may use the second, minute, hour, day or year. To measure weight we use the ounce, pound or ton. In every case and for every quantity that we want to measure we must have first of all *definite*, *simple*, and *permanent* units of measure.

Study carefully the following tables and use them to work out the problems which follow.

2

LINEAR OR LONG MEASURE

12	inches (in.)	make	1 foot	(ft.)
3	feet	make	1 yard	(yd.)
5½	yards	make	1 rod	(rd.)
320	rods	make	1 mile	(mi.)
5280	feet	make	1 mile	

SQUARE MEASURE

144	square inches (sq. in.)	make	1 square foot	(sq. ft.)
9	square feet	make	1 square yard	(sq. yd.)
30¼	square yards	make	1 square rod	(sq. rd.)
160	square rods	make	1 acre	(A.)
640	acres	make	1 square mile	(sq. mi.)

CUBIC MEASURE

Used to compute the contents or volume of objects.

1728	cubic inches (cu. in.)	make	1 cubic foot	(cu. ft.)
27	cubic feet	make	1 cubic yard	(cu. yd.)
128	cubic feet	make	1 cord of wood	(a pile 4×4×8 ft.)
7.48	U. S. gallons (gal.)	make	1 cubic foot.	

24¾ cubic feet make 1 perch of masonry (16½×1½×1 ft.)

TIME MEASURE

60	seconds (sec.) or ″	make	1 minute	(min.) or ′
60	minutes	make	1 hour	(hr.)
24	hours	make	1 day	(da.)
7	days	make	1 week	(wk.)
365¼	days	make	1 year	(yr.)

AVOIRDUPOIS WEIGHT

Used to weigh all metals except gold and silver.

16	drams (dr.)	make	1 ounce	(oz.)
16	ounces	make	1 pound	(lb.)
25	pounds	make	1 quarter	(qr.)
4	quarters	make	1 hundredweight	(cwt.)
20	hundredweight	make	1 short ton	(T.)
2240	pounds	make	1 long ton	

1 stone = 14 pounds   1 quintal = 100 pounds

WEIGHTS OF MATERIALS IN POUNDS PER CUBIC FOOT

Steel	489.6	Brick, soft	100
Cast iron	450	Brick, hard	125
Wrought iron	480	Brick, fire	150
Copper	552	Cement, Portland	78
Brass	522	Clay	135
Bronze	552	Concrete	130
Aluminum	166.5	Earth, loose	76
Lead	709.7	Sand	100
Nickel	548.7	Stone, various	135–200
Zinc	436.5	Wood, pine, white	28
Coal, hard	60	Wood, oak, white	48
Coal, soft	53	Wood, hickory	48
Marble	168.7	Air	0.080723
Cork	15		

1 cu. ft. of water weighs 62.5 lb.
1 gal. of water weighs 8.345 lb.

UNITED STATES DRY MEASURE

2 pints = 1 quart
8 quarts = 1 peck
4 pecks = 1 bushel

The Winchester bushel which is a cylinder 18½ in. diam. and 8 in. deep is the standard U. S. bushel. It contains 2150.42 cu. in.

CIRCULAR MEASURE

60 seconds (″) = 1 minute (′)
60 minutes (′) = 1 degree (°)
90 degrees = 1 quadrant
360 degrees = 1 circumference

12. Explanation of Types of Locomotives.—In the work and problems which follow in this book reference is made to different types of locomotives, such as Pacific, Atlantic, Mallet, and so forth. The following description is therefore given of a few of the more common types by way of explanation.

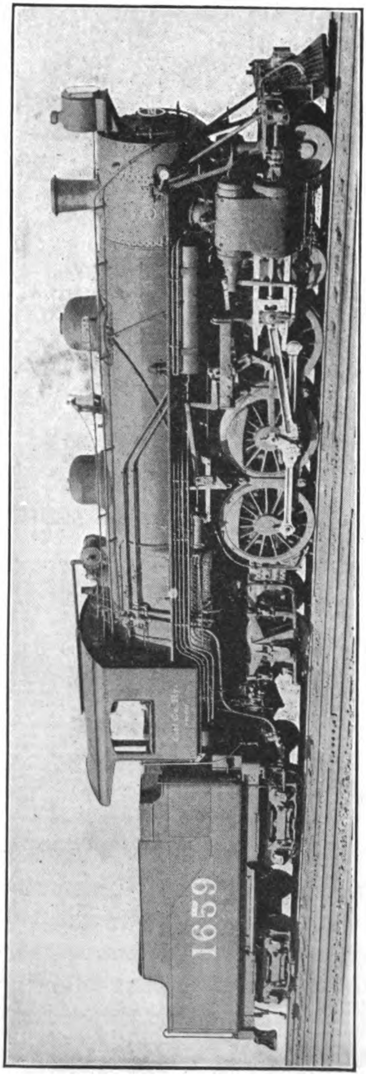


Fig. 5.—Pacific type locomotive.

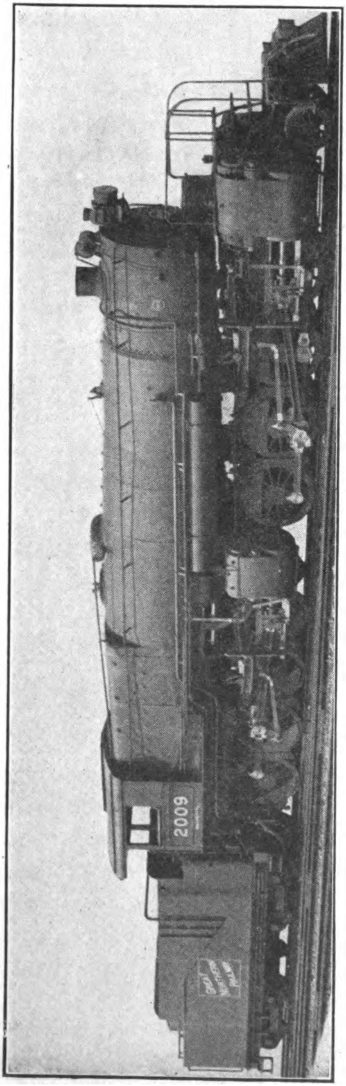


Fig. 6.—Mallet type locomotive.

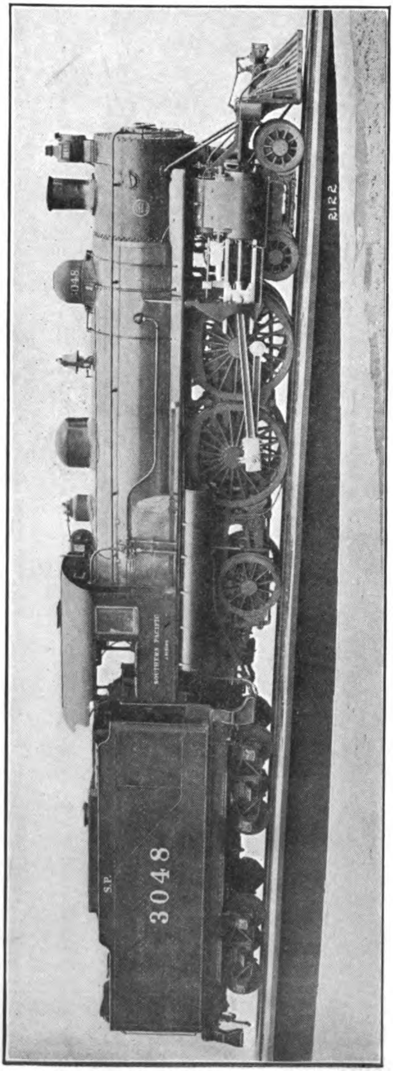


FIG. 7.—Atlantic type locomotive.

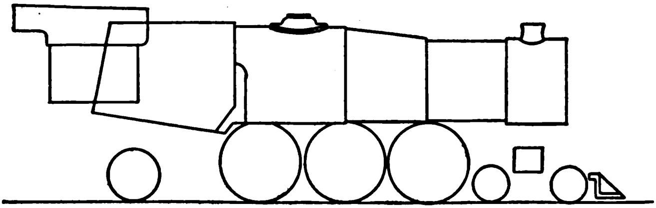


FIG. 8.—Pacific type locomotive. Four leading truck wheels, six drivers and two trailing truck wheels.

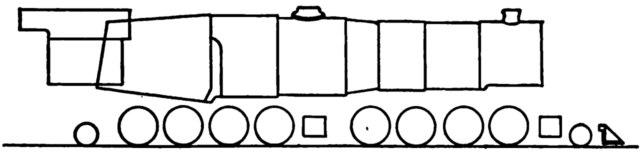


FIG. 9.—Mallet type locomotive. Two leading truck wheels, sixteen drivers, and two trailing truck wheels.

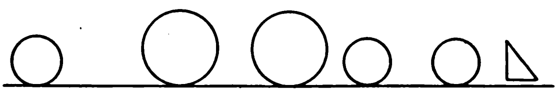


FIG. 10.—Atlantic type locomotive. Four leading truck wheels, four drivers, and two trailing truck wheels.

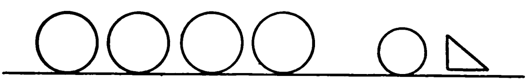


FIG. 11.—Consolidation type locomotive. Two leading truck wheels, eight drivers, and no trailing truck wheels.

Diagrams showing wheel arrangement of various types of locomotives.

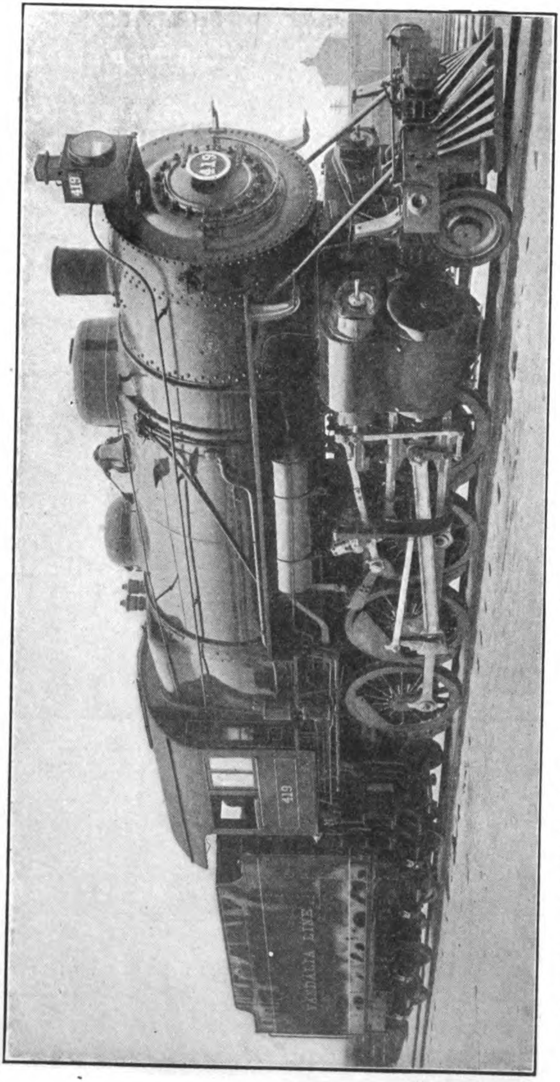


FIG. 12.—Consolidation type locomotive. Used for freight service on the Pennsylvania Lines.

Figure 5, page 20, shows a Pacific type locomotive which has six driving wheels, four wheels in the leading truck and two in the trailing truck. This type of locomotive is used by a large number of railroads for passenger service and is known by all as the Pacific type.

Figure 6 shows a Mallet four-cylinder, sixteen driver locomotive. There are two wheels in the leading truck and there is no trailing truck. This type of locomotive is used for freight service on account of the heavy pull which it can exert.

The Atlantic type of locomotive Fig. 7 has four wheels in the leading truck, four driving wheels, and two wheels in the trailing truck. Figs. 8, 9, 10, and 11 show the wheel arrangement of the various types of locomotives.

PROBLEMS FOR PRACTICE

17. How many eighths are there in $3\frac{5}{8}$ in.? In $4\frac{3}{8}$ in.? In $7\frac{5}{8}$ in.?

18. Reduce to improper fractions, $5\frac{9}{16}$ in., $7\frac{5}{16}$ in., $9\frac{7}{8}$ in., $4\frac{13}{32}$ in.

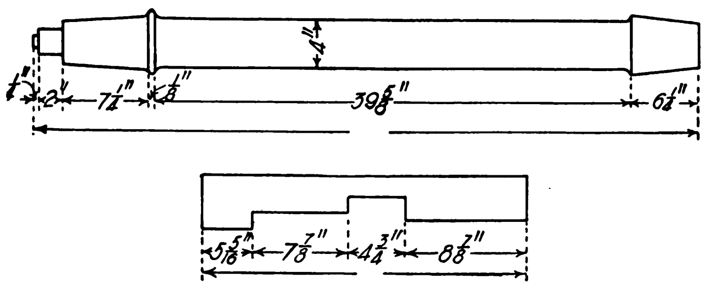


FIG. 13.

19. Reduce $\frac{5}{2}$, $\frac{7}{3}$, $\frac{19}{5}$, and $\frac{17}{4}$ to mixed numbers. (Divide the denominator into the numerator and place to the right of the quotient obtained the remainder written above the denominator.)

Mixed numbers may be added by adding to the sum of the whole numbers the sum of the fractions.

20. A drawing calls for the following divisions: $5\frac{5}{16}$ in., $7\frac{7}{8}$ in., $4\frac{3}{4}$ in., and $8\frac{7}{8}$ in. Find the overall dimensions.

21. (*a*) Find the overall dimension of the piston rod shown in Fig. 13, page 25.

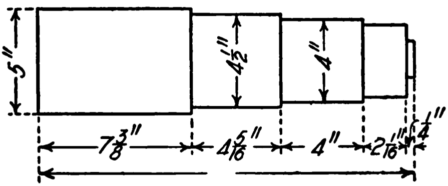


FIG. 14.

(*b*) If $\frac{1}{4}$ in. is allowed at each end for finishing what must be the length of the rough forging for the piston rod?

22. A crank pin has the dimensions as shown in Fig. 14. How long must be the rough forging so that after taking $\frac{1}{4}$ in. off at each end for finishing it will have the length given?

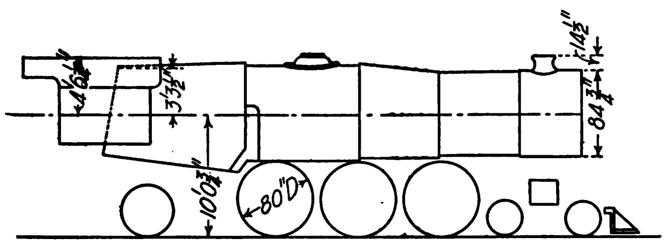


FIG. 15.

23. A casting weighs $14\frac{7}{8}$ lb. in the rough. How much will it weigh after machining if $3\frac{5}{16}$ lb. are turned off? (Subtract fractions first and then the whole numbers.)

24. The distance from center to center of two holes, respectively $1\frac{7}{8}$ in. and $1\frac{5}{8}$ in. in diameter, is $3\frac{3}{16}$ in. How much metal is left between the holes?

25. Four pieces are cut from a ½-in. round iron rod, one 3 ft. 3½ in., one 5 ft. 6⅜ in., one 2 ft. 7 5/16 in. and one 13 in. long. If the original piece was 16 ft. long how long a piece remains?

26. In Fig. 15, showing a Pacific type locomotive, find:
(1) The height of top of cab above rails.
(2) The height of rear end of boiler above rails.
(3) The height of top of stack above center line of boiler.
(4) The height of center line of boiler above center of drivers.

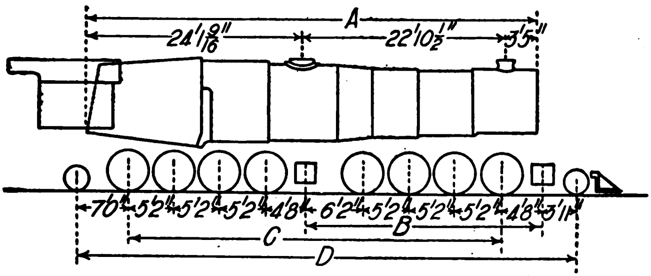


FIG. 16.

27. In Fig. 16, showing a Mallet type locomotive, find:
"*A*" The overall length of the boiler.
"*B*" The length between cylinders.
"*C*" The length between first and last drivers.
"*D*" The length between centers of leading and trailing truck wheels.

In multiplying or dividing mixed numbers reduce them to improper fractions and proceed as in ordinary fractions.

28. Multiply 7⅛ by 2½, 4⅝ by 7 9/16, 8¾ by 6⅔, 4¾ by 7½.

29. Which would be the cheaper, to employ a man at 21½ cts. an hr. who takes 6¾ hr. to do a piece of work or to employ a man at 18½ cts. an hr. who takes 7¼ hr. to do the same work?

30. At 5⅝ cts. a lb., find the cost of a casting weighing 17½ lb.

31. The parts of a motor driving a boring mill weigh as follows: Frame 103½ lb., end plates 12⅓ lb., bed plate 57¾ lb., shaft 16 lb., and resistance rings $9\frac{1}{2}$ lb. What is the total weight of the parts mentioned?

32. What is the average weight of the following castings: first, $49\frac{1}{2}$ lb.; second, $56\frac{1}{2}$ lb.; third, $39\frac{1}{4}$ lb.; and fourth, $137\frac{3}{4}$ lb.?

33. A barrel contains $45\frac{1}{2}$ gal. of oil. If $24\frac{3}{4}$ gal. are taken out at one time and $11\frac{3}{8}$ gal. at another time, how many gallons are left?

34. In an engine test the horsepower at intervals of 5 min. is found to be $63\frac{1}{4}$, $62\frac{1}{8}$, $60\frac{3}{4}$, $59\frac{2}{3}$, $64\frac{3}{4}$, and $61\frac{1}{2}$. Find the average horsepower.

35. There are three castings weighing respectively 460 lb., 134 lb., and 78 lb. each. The first loses $\frac{1}{4}$, the second $\frac{1}{5}$ and the third $\frac{1}{6}$ of its weight due to machining. Find the total weight of the three finished pieces.

36. If it takes $1\frac{1}{4}$ lb. of babbitt for one bearing, how many pounds will be required for 235 bearings?

37. If the cost of repairs on the boiler of a locomotive is $420, which is found to be $\frac{2}{3}$ of the entire cost of all the repairs for the locomotive, what is the total cost of the repairs?

Solution:

If $\frac{2}{3}$ of entire cost is $420

then $\frac{1}{3}$ of entire cost is only $\frac{1}{2}$ of $420 or $210

and $\frac{3}{3}$ or the entire cost is 3 times $210 or $630. *Ans.*

38. The cost of repairs on the cylinder of a locomotive is $45 which is $\frac{5}{8}$ of the cost of repairs on the frame. Find the cost of repairs on the frame.

39. A firm sold a machine for $600 which was $\frac{3}{4}$ of the cost. Did it gain or lose on the transaction, and how much?

40. A locomotive has a total heating surface of 3925 sq. ft., $\frac{3}{25}$ of this amount being in the fire box and the remainder in the tubes. Find the number of square feet in each.

41. An axle weighs 1024 lb. in the rough. If $\frac{5}{64}$ of this weight is turned off, what is the weight of the finished axle?

42. From a lot of brass castings weighing 480 lb. how many lugs can be cast each weighing $\frac{3}{4}$ lb., providing there is no loss of metal?

CHAPTER III

DECIMALS

13. Definitions.—Fractions that have 10 or some multiple of 10 (that is, 100, 1000, etc.) as a common denominator are called decimal fractions.

Thus $\frac{1}{10}$, $\frac{5}{100}$, $\frac{625}{1000}$ are decimal fractions. Decimal fractions are usually written without the denominator; thus, $\frac{1}{10} = .1$, $\frac{5}{100} = .05$, and $\frac{625}{1000} = .625$.

A decimal fraction when written without a denominator is called a decimal. In the above examples we see that the number of "decimal places" or figures to the right of the decimal point is equal to the number of ciphers in the denominator when the decimal is written as a fraction. Decimals are always read the same as if they were written as fractions; thus, .07 ($\frac{7}{100}$) is read seven one-hundredths. Decimals whose values are less than one are called pure decimals. If a whole number and a decimal are written together, as 6.25, the combination is called a mixed decimal, and is read 6 and 25 one-hundredths. The whole number is read exactly the same as an ordinary number. For the decimal point we may say "and," after which we read the decimal like an ordinary number and give it the denominator of the fraction which it should have.

The following table gives the decimal scale of notation as well as that of the whole numbers.

Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Units	Decimal point	Tenths	Hundredths	Thousandths	Ten thousandths	Hundred thousandths	Millionths
7,	8	5	6,	0	3	0	.	7	0	9,	8	6	0

United States money, for example, is written in decimals. The first place to the right or the "tenths" place can represent "dimes" since a dime is $1/10$ of a dollar. The second or "hundredths" place would indicate "cents" since a cent is $1/100$ of a dollar. The third or "thousandths" place would indicate "mills" since one mill is $1/1000$ of a dollar. We thus see that as we advance toward the right the value of each space is $1/10$ of the one directly to the left of it.

14. Addition and Subtraction.—If we wish to add dollars and cents we must be careful to add dollars to dollars and cents to cents; thus, if we wish to find the sum of $2.15, $7.85, $5 and $6.20, we must place cents below cents, etc.; or in other words, if we wish to add decimals we must *keep the decimal point in a vertical line*, and we find that the sum is $21.20:

```
  $2.15
   7.85
   5.
   6.20
 ------
 $21.20
```

The same holds true in the subtraction of decimals.

PROBLEMS

1. Add six dollars and seventy-five cents, three dollars and fifty-one cents, two dollars and three cents, and seven dollars.

2. A machine was bought for $75.25 and sold for $100. What was the gain?

3. The correct size of a reamer is .500 in., but the tool maker finds that it is only .499 in., how much too small is it?

4. A reamer is 1.25 in. in diam. on the small end and 1.375 in. on the large end. Find the difference in diameter of ends of reamer.

5. A foreman gave a tool maker three distance blocks to make a snap gauge that would just fit when the three blocks were put together. The blocks were .375 in., .125 in., and .5625 in. What was the size of the snap gauge?

6. Write the following using the dollar mark ($): 16 cts., 1½ cts., 136 cts., 385 cts.

7. Using the cent mark (¢) write $3.75, $8.455, $7.63, $845.

15. Reducing Fractions to Decimals.—If we wish to use a micrometer caliper or a scale with 10th and 100th divisions, we see at once that ordinary fractions cannot be used. In this case we must reduce them to decimals, which can be done by placing a decimal point after the numerator, adding as many ciphers as required, and dividing it by the denominator.

Thus:

$$\begin{array}{r} \frac{1}{4})\underline{1.00} \\ .25 \end{array}$$

Hence ¼ = 25 hundredths.

After dividing we must point off as many places to the left as we added ciphers after the decimal point. Usually we add ciphers until the result comes out even. In many cases, however, the result will never come out even as is the case if we wish to reduce ⅓ to a decimal.

In this case if we would consider hundredths close enough, we would add two ciphers, thus:

$$3\overline{)1.00}$$
$$.33+$$

The plus (+) sign written after the result indicates that the correct answer would be slightly more. We seldom work closer than the thousandths place, hence three ciphers added would be sufficient.

PROBLEMS

8. To what reading would you set a micrometer caliper to test the following sizes of taps: 3/8 in., 5/8 in. and 7/8 in.?

9. Find the decimal equivalents of fractions of an inch going by 32nds; get results correct to four figures and tabulate them thus:

Fraction	Decimal	Fraction	Decimal
1/32	.03125	17/32	.5313
1/16	.0625	9/16	.5625

Keep this table for future reference.

16. Reducing Decimals to Fractions.—Decimals may be reduced to ordinary fractions by omitting the decimal point and writing them as decimal fractions, with the proper denominator, and then reducing to lowest terms, thus:

$$.375 \text{ in.} = 5\left|\frac{375}{1000}\right. = 5\left|\frac{75}{200}\right. = 5\left|\frac{15}{40}\right. = \frac{3}{8} \text{ in.}$$

17. Multiplication of Decimals.—Decimals may be multiplied the same as whole numbers, providing we point off as many places in the answer as we have decimal places in both numbers. For example, multiply .25 by .35:

$$\begin{array}{r} .25 \\ .35 \\ \hline 125 \\ 75 \\ \hline .0875 \end{array}$$

that is, .25 × .35 = .0875 or eight hundred seventy-five ten thousandths.

PROBLEMS

10. Find the weight in lb. per cu. ft. of the following metals (1 cu. ft. = 1728 cu. in.):

Metal	Wt. per cu. in. in lb.	Wt. per cu. ft. in lb.
Wrought iron	.28	
Steel	.283	
Cast iron	.261	

11. A cubic inch of cast iron weighs .26 lb. What is the weight of a casting that contains 375.35 cu. in.? Of one that contains 5 cu. ft.?

12. A cubic inch of steel weighs .283 lb. What is the weight of a steel shaft containing 356 cu. in.?

13. A locomotive weighs 238,500 lb. If .752 of this weight is on the drivers, find the weight on the drivers. Find the weight on the trucks.

14. The total heating surface of the boiler on a locomotive is 4500 sq. ft.; if .4509 of this is in the fire box, find the number of square feet of heating surface in the fire box.

15. If the tractive power of a locomotive per lb. of mean effective pressure in the cylinder is 187.2 lb., what is the total tractive force with a mean effective pressure of 161.4 lb.?

18. Division of Decimals.—We divide decimals the same as whole numbers and point off as many places in the answer, or quotient, as the number of places in the dividend (or the number divided) exceed those in the divisor. In case there are no places in the dividend we must add ciphers so that the number of places in the dividend will at least be equal to those in the divisor, thus:

Divide 1.6524 by .027.

```
.027)1.6524(61.2
     1 62
     ----
        32
        27
        --
         54
         54
         --
```

The divisor (.027) has three places and the dividend (1.6524) has four places; therefore the quotient has 4 minus 3 or 1 decimal place and is 61.2. In case the quotient does not come out even we may annex ciphers to the dividend and continue dividing until we have the desired number of figures in the answer. Usually three figures are close enough.

If we wish to divide a decimal by 10 or any multiple of 10 we move the decimal point as many places to the *left* as there are ciphers in the divisor; thus, 387 ÷ 1000 = .387. In multiplying by 10 or any multiple of ten, move the decimal point to the *right* as many places as there are ciphers in the multiplier; thus, 4.326 × 100 = 432.6.

PROBLEMS

16. A locomotive used 27.04 tons of coal in running 310.08 miles. What was the coal consumed per mile?

17. If the weight of a certain size of bolts is 69.3 lb. per 100 bolts, what is the weight per bolt?

18. Divide 8967 by 45.8 to 3 decimal places.
Divide 6785 by 3.1416 to 2 decimal places.
Divide 6872 by 14.7 to 1 decimal place.
Divide 876.4 by 32.2 to 2 decimal places.

MISCELLANEOUS PROBLEMS

19. The weight of a locomotive is distributed as follows:

Wt. on truck	35,000 lb.
Wt. on 1st pair of drivers	55,000 lb.
Wt. on 2nd pair of drivers	56,600 lb.
Wt. on trailing wheels	31,200 lb.

Find total weight of engine.

20. The inside diameter of a cylinder before boring is 24 in.; after boring it is 24.375 in. How deep a cut was taken?

21. A micrometer caliper shows a piece to have a diameter of .873 in. What would this diameter be when expressed in the nearest 64th of an inch? (Multiply .873 by 64.)

22. Reduce the following decimals of an inch to the nearest 16th of an inch:

.875	.125	.625
.386	.1875	.6875
.8125	.0625	.375

(Multiply each decimal by 16.)

23. If $\frac{4}{5}$ of a job is done in 24 min., in what decimal part of an hour would $\frac{1}{2}$ the job be done?

24. A loaded freight car weighs 142,500 lb. If $\frac{3}{10}$ of this is the weight of the car what is the weight of the load carried?

25. If the weight of 156 similar steel bars each 9 ft. long is 2808 lb., what is the weight per ft. of bar?

26. The distance between centers of two holes is $3\frac{1}{2}$ in. If the holes are $\frac{15}{16}$ in. in diam., what length of metal is left between them?

27. To what part of the actual size is a drawing made when drawn to a scale of 3 in. = 1 ft. 0 in.; of $1\frac{1}{2}$ in. = 1 ft. 0 in.; of $\frac{3}{4}$ in. = 1 ft. 0 in.; of $\frac{1}{8}$ in. = 1 ft. 0 in.

28. A tool has a feed of 8 cuts to the inch. How many cuts must be made to plane a piece 2 ft. 6 in. wide?

29. If the time to make 1 cut (forward and return motion of planer table) is 45 sec., how long will it take to finish the job given in problem 28?

30. If it requires 5 men to complete a job in 6 days, how long would it take if 8 men are put on the work?

31. From a ton of iron bars each weighing 80 lb. 3/8 of this is cut up for bolts, 1/8 for shafts and the remainder for studs. How many bars are used for the different articles?

32. How many times will the table of a boring mill turn in truing up if the tool has a feed of 1/4 in. and the table is 42 in. in diam.?

19. The Micrometer.—The micrometer, in the form of a micrometer caliper as shown in Fig. 17, is used to measure to the thousandth part of an inch. The instrument is made up of parts as shown in the figure. The principle of the micrometer is as follows: The spindle is threaded inside of the sleeve with 40 threads to the inch. The thimble is attached to the end of the spindle. The piece to be measured is placed between the spindle and anvil. The caliper is then closed on it by turning up the thimble. Since the screw on the spindle has 40 threads to the inch its pitch is 1/40 in. or .025 in., so that one turn of the thimble varies the opening 1/40 or .025 in. Each mark on the barrel or sleeve represents one complete turn of the thimble. Four turns therefore equal 4 × 1/40 or 1/10 in. Every fourth mark is therefore extended and numbered 1, 2, 3, etc., representing tenths of an inch. The outside thimble is divided into 25 equal divisions so that turning the thimble one division turns the screw 1/25 of a complete turn and advances it 1/25 of 1/40 or 1/1000 in. Each of these small divisions therefore represents 1/1000 or .001 in. To read the micrometer, set down first the number of tenths of an inch as shown by the last number exposed on the sleeve. Next count the number of small divisions on the sleeve exposed between the last tenths division and the edge of the barrel. Multiply this number by .025 and add to the number of tenths. Lastly add the number of spaces on the edge of the thimble that it has been turned from its zero point. This last reading is in thousandths of an inch and should be added to the reading already taken. This gives the complete reading in

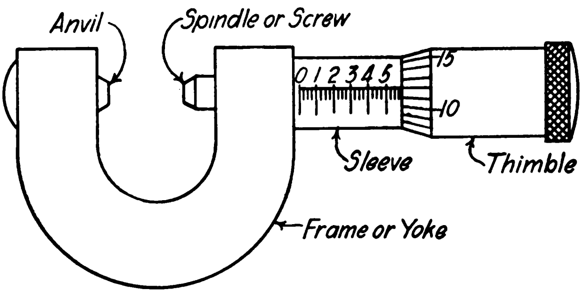


FIG. 17.

thousandths of an inch. The micrometer in Fig. 17 reads as follows:

5 large divisions on the sleeve	=	.5 in.
3 divisions between the .5 mark and the edge of the thimble = 3 × .025	=	.075
12 divisions on the edge of the thimble	=	.012
Total reading	=	.587 in.

or five hundred eighty-seven thousandths of an inch.

The following readings in Figs. 18 and 19 are worked out by way of further explanation.

By estimating tenths of spaces on the thimble we may approximate the tenthousandths of an inch.

For example, in Fig. 20, the reading is .3834 in.;

$$\begin{array}{l} .3 \text{ in.} \\ .075 \text{ in.} \\ \underline{.008\ (4)} \text{ in. (estimated)} \\ .3834 \text{ in.} \end{array}$$

For convenience most micrometers have stamped in the yoke or frame the decimal equivalents of the frac-

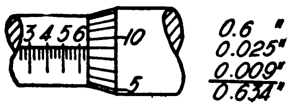


FIG. 18.

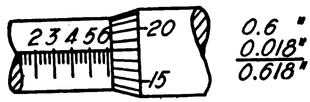


FIG. 19.

tions of an inch by sixty-fourths from $\frac{1}{64}$ to 1 in. It is well to remember that $\frac{1}{8}$ in. = .125 in. Therefore $\frac{3}{8} = 3 \times .125$ or .375 in. and $\frac{1}{16} = \frac{1}{2}$ of $\frac{1}{8} = \frac{1}{2} \times .125 = .0625$ in.; then $\frac{3}{16} = 3 \times .0625 = .1875$ in. and so on.

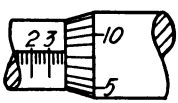


FIG. 20.

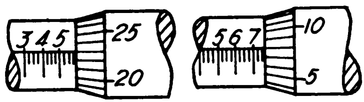


FIG. 21.

Since an increasing number of measurements in the shop are being made with the micrometer it is well to become thoroughly familiar with its use.

Take the micrometer readings in Fig. 21.

Decimal Equivalents of Fractions of an Inch. (Advancing by 64ths.)

1/64 = .015625	17/64 = .265625	33/64 = .515625	49/64 = .765625
1/32 = .03125	9/32 = .28125	17/32 = .53125	25/32 = .78125
3/64 = .046875	19/64 = .296875	35/64 = .546875	51/64 = .796875
1/16 = .0625	5/16 = .3125	9/16 = .5625	13/16 = .8125
5/64 = .078125	21/64 = .328125	37/64 = .578125	53/64 = .828125
3/32 = .09375	11/32 = .34375	19/32 = .59375	27/32 = .84375
7/64 = .109375	23/64 = .359375	39/64 = .609375	55/64 = .859375
1/8 = .125	3/8 = .375	5/8 = .625	7/8 = .875
9/64 = .140625	25/64 = .390625	41/64 = .640625	57/64 = .890625
5/32 = .15625	13/32 = .40625	21/32 = .65625	29/32 = .90625
11/64 = .171875	27/64 = .421875	43/64 = .671875	59/64 = .921875
3/16 = .1875	7/16 = .4375	11/16 = .6875	15/16 = .9375
13/64 = .203125	29/64 = .453125	45/64 = .703125	61/64 = .953125
7/32 = .21875	15/32 = .46875	23/32 = .71875	31/32 = .96875
15/64 = .234375	31/64 = .484375	47/64 = .734375	63/64 = .984375
1/4 = .25	1/2 = .50	3/4 = .75	

CHAPTER IV

PERCENTAGE

20. Definitions.—Percentage is a term applied to calculations taking 100 as the base. It is closely related to decimals.

Per cent. means "by the hundred" and is written thus, "%."

One per cent. (1%) would be 1 out of every hundred or $\frac{1}{100}$ of whatever we are talking about.

For example, a certain shop employing 200 men increases its number of men 10%. How many men will it have after increasing its force? Here 10% = $\frac{10}{100}$ or $\frac{1}{10}$ of 200 or 20 men, and the number of men employed is 200 + 20 or 220 men. Here 200 is taken as the *base* and is of course 100%. 10% is called the *rate* per cent., and 20 is called the *percentage* and 220 the *amount*.

21. To Find the Percentage and the Amount.—In finding the percentage, we write the rate per cent. as hundredths and multiply it by the base or the number of which we wish to find the percentage.

PROBLEMS

1. The interest on an investment is often taken as 5%. At this rate what would be the interest on the investment for a shop (buildings only) valued at $1,927,003?

2. The valuation of the tools and machinery for a shop is $2,192,773. If 5% is allowed for depreciation what would this depreciation be?

The efficiency of a motor or an engine is taken as the percentage of the work supplied, that is, given out at the shaft. If a motor is 80% efficient it means that out of 100 h.p. of the power supplied or furnished to the motor, 80 h.p. is available at the shaft for driving machinery and that 20 h.p. is lost, due to friction, etc.

3. An engine rated at 1250 h.p. is found to be 85% efficient. How many h.p. are available for driving machinery? How much is lost?

4. A firm sells a locomotive which cost them $15,000 at a gain of 15%. What is their gain? What is the selling price?

5. What is

10% of 50	50% of 960
18% of 72	25% of 830
2½% of 64	37½% of 720
19¾% of 24	18¾% of 32
6% of 78,240	

6. A carload of pig iron weighs 90,000 lb. If 11⅙% of this is used at once in the foundry, how much is left?

7. If out of 720 car wheels 5% are rejected on account of defects, how many are rejected? How many are left? What per cent. are left?

8. A firm built two locomotives, the first costing $15,000 and the second $12,000. The first was sold at a loss of 2% and the second at a gain of 2½%. Did the firm gain or lose on the transaction?

9. A firm sold two machines, losing 15% on the first. If the first cost $300 and the second $250, for how much must the second machine be sold so that the firm would gain $50 on the whole transaction?

10. A locomotive has a piston displacement of 12,656 cu. in. If the clearance space is 6.5% of the piston displacement, what is the clearance space?

11. Three men are paid as follows: 20 cts., 25 cts. and 30 cts. an hr. If each is given a raise of 10%, how much will each get after the raise?

12. An axle originally weighs 1250 lb. and loses 6% of its weight in turning. What is the weight after turning?

13. If the braking power is 89% of the weight of a car, find the braking power of a car weighing 120,600 lb.

14. Out of 758,000 ties 14% were treated with creosote and the remainder were untreated; find the number of each.

15. If creosoting increases the life of a tie 45%, how long would a tie last when creosoted if it would last 8 yr. when untreated?

16. If the average cost of transportation by rail in the U. S. was 5 mills per ton mile in 1907 and in 1908 it was found to be 20% more, what was the rate in 1908?

17. A train of 150 cars is being hauled by three locomotives. The first locomotive hauls 20% of the whole train, the second 30%, and the third the remainder. How many cars were drawn by each locomotive?

18. The grade of a railroad track is expressed in per cent. A rise of 1 ft. in 100 ft. is called 1% grade. What would be the difference in height of the two ends of a track 3 miles in length with a 1½% grade?

19. In order that the piston rings exert an outward pressure on the cylinder so as to prevent leakage of steam past the piston, they are usually turned to a diameter 1½% larger than the piston diameter. To what diameter (nearest 32nd) would you turn the piston rings for the following sizes of pistons: 15 in., 16 in., 18 in., 20 in., 21 in., 22 in., 24 in. and 26 in. diam.?

22. To Find the Base.—If we have given the percentage and the rate per cent, we divide the percentage by the rate written as hundredths and the result will be the base.

Thus, $21.60 is 6% of how many dollars?

$$6\% = .06$$
$$\$21.60 \div .06 = \$360 \text{ or the base.}$$

PROBLEMS

20. The interest on the investment at 5% for a shop is $96,350.15. What is the valuation of the shop?

21. The weight on the drivers of a locomotive is 178,500 lb. which is 65.62% of the total weight. What is the total weight of the locomotive?

22. If 3187.5 h.p. are required to drive a machine shop, what would be the horsepower intake of an engine driving the machines if the efficiency of the engine is 85%?

23. If from a steel plate 75 sq. in. are punched out, which is 15% of the total area, what was the area of the plate before punching?

24. If from a rough axle 75 lb. have been turned off, which is 6% of the weight of the rough axle, find the weight of the axle before and after turning.

25. If the difference in level between the bottom and top of a 1½% grade is 79.2 ft., find the length of track having the 1½% grade.

26. The total braking power on a passenger car is 80,000 lb. which is 90% of the total weight of the car. Find the weight of the car.

27. If the cylinder clearance of a locomotive is 822.64 cu. in., which is 6½% of the piston displacement, what is the piston displacement?

28. A man's pay was increased 30 cts. a day which was 10% of his original pay. How much was he getting before his increase? How much is he getting now? If he gets a cut of 10%, how much will he be getting after the reduction? Why is a 10% cut more than a 10% increase in this particular case?

29. $60 is 15% of how many dollars? 750 bolts are 18¾% of how many bolts? 250 cars are 8% of how many cars?

30. If the tractive force or pull on the draw bar of a locomotive is 20% of the weight on the drivers, what is the weight on the drivers of a locomotive having a tractive force of 37,600 lb.?

31. A man gained $50 on a machine, which was 20% of the cost. Find the cost and the selling price.

23. To Find the Base when We Have the Rate and the Amount.—If we have given the amount and the rate and desire to find the base, we add the rate written as hundredths to 1.00 and divide it into the amount.

Thus, what sum will at 6% amount to $381.60?

$$6\% = .06 + 1.00 = 1.06$$
$$\$381.60 \div 1.06 = \$360$$

PROBLEMS

32. If the interest at 5% on a certain sum added to the sum itself amounts to $525, what is the sum and what is the interest?

33. The load on a car after being increased 60% is 96,000 lb. What was the load before the increase?

34. A man's pay after a 10% increase is $3.30. What was his pay before the increase and what was the increase?

35. The efficiency of an engine is 85%. What must be the rated horsepower of the engine in order to have 1700 h.p. available to drive machinery? (Divide the horsepower available at the shaft by the efficiency.)

24. To Find the Rate when We Have the Percentage and the Base.—To find the rate per cent. having given the base and the percentage divide the percentage by the base and the result will be the rate expressed in hundredths.

Thus, at what rate will $360 give $21.60 interest?

$$\$21.60 \div 360 = .06 \text{ or } 6\%$$

PROBLEMS

36. A locomotive weighing 154,700 lb. has 101,550 lb. on the drivers. What per cent. of the total weight is on the drivers? What per cent. is on the trucks?

37. What per cent. of 60 is 6? What per cent. of 72 is 8? What per cent. of 96 is 16? What per cent. of 125 is 25?

38. A man's pay is increased from $3 to $3.30. What is the rate of increase?

39. Out of 300 h.p. supplied to a motor 240 are available for driving machinery. What is the efficiency of the motor?

40. Out of 5760 rivets heated, 438 are burned. What per cent. of the rivets are burned?

41. The area of a steel plate before punching is 28 sq. ft. and after punching it is only 21 sq. ft. What per cent. of the metal is punched out? What per cent. is left?

42. Find the per cent. of the total weight on each pair of drivers of the following locomotives:

Classification	Freight	Passenger
Weight of engine, pounds............	186,500	317,000
Weight on 1st pair of drivers.........	43,800	66,000
Weight on 2nd pair of drivers........	40,200	65,800
Weight on 3rd pair of drivers.........	41,500	66,000
Weight on 4th pair of drivers.........	40,900	....

MISCELLANEOUS PROBLEMS ON PERCENTAGE

43. The weight of a passenger locomotive is distributed as follows:

Weight on truck.........................	47,500 lb.
Weight on 1st pair of drivers................	55,500 lb.
Weight on 2nd pair of drivers...............	62,000 lb.
Weight on 3rd pair of drivers...............	61,000 lb.
Weight on trailer.........................	46,000 lb.

What per cent. of the total weight is on the truck? On the drivers (three pairs)? On the trailer? (Find percentage to two decimal places.)

44. The repairs on the cylinders of an engine cost $24 which was 20% of the entire cost of repairs. Find the total cost of the repairs.

45. A firm sold two machines for $360 each, gaining 20% on the first and losing 20% on the second. Did the firm gain or lose and how much?

46. Find the per cent. of grade in each of the following: 35 ft. rise per mile; 75 ft. rise per mile; 48.2 ft. rise per mile; 86.3 ft. rise per mile.

47. A rack contains 7856 lb. of wrought-iron bars. If 35% is used on one stock order, 25% is used on another stock order, and 75% of what remains on a third stock order, how many pounds remain?

48. If we take a contract for $2000 and it costs us $1600, what percentage have we gained on the cost?

49. The reverse lever on a locomotive is placed so that cut-off takes place when the piston has moved 6½ in. from the beginning of the stroke. At what per cent. of the stroke does cut-off occur if the whole stroke is 26 in.?

50. If the drivers on a locomotive slip 3 revolutions in 75, what is the percentage of slip?

51. A bed plate of an engine is being turned out in the shop. A machinist at 45 cts. an hr. takes 5 hr. to place it and uses 2 hr. of a helper's time whose pay is 20 cts. an hr. Another machinist at 30 cts. an hr. takes 3 hr. to drill all holes, etc. An apprentice at 18 cts. an hr. takes 3 hr. to paint it. Two dollars are allowed for incidentals. It weighs 960 lb. and the material costs 6½ cts. per lb. Find the total cost of the bed plate. What must it be sold for in order to gain 12½% on the cost price?

52. A raise of 6% is declared on piecework rates. What would be the new rates for operations which paid the following amounts: $3.75, $1.25, $.675 and $.35?

53. Out of 3576 rivets heated, 150 are burned. What per cent. of the total amount heated are burned? What per cent. are used?

CHAPTER V

RATIO AND PROPORTION

25. Definitions, Ratio.—A ratio deals with two numbers or quantities and indicates how many times one quantity is greater than the other. Thus, the ratio of 8 to 4 is 2, meaning that 8 is twice 4. The ratio of 12 to 4 is 3, etc. Ratio is expressed the same as division. Thus if we have one rack containing 1500 lb. of iron bars and another rack containing 750 lb., the ratio of the weights in the first and second racks would be 1500 ÷ 750 or 2, meaning that the first rack contains twice as much as the second.

A ratio is usually written thus, 1500 : 750; that is, the division or ratio is indicated by means of two dots instead of the regular division sign (÷). A ratio may also be written as a fraction; thus the ratio of 700 to 1750 is $\frac{700}{1750} = \frac{2}{5}$.

PROBLEMS

1. What is the ratio of the number of teeth of the driver to the number of teeth of the driven in the following sets of gears:

1st set, driver 30 teeth.............driven 60
2nd set, driver 80 teeth.............driven 50
3rd set, driver 120 teeth............driven 30

2. A locomotive has a heating surface of 1341 sq. ft. and a grate area of 22.87 sq. ft. What is the ratio of grate area to the heating surface? (The ratio of heating surface to grate area equals heating surface divided by grate area and the ratio of grate area to heating surface equals grate area divided by heating surface, both areas being taken in the *same units*.)

3. The diameter of the cylinder on an engine is 20 in. and the diameter of the piston rod is 3 in. What is the ratio of the cylinder diameter to the piston rod diameter, that is, how many times larger is the cylinder diameter than the piston rod diameter?

4. The steam pressure in a locomotive is 205 lb. and the mean effective pressure in the cylinders is found to be 85 lb. What is the ratio of the mean effective pressure to the boiler pressure?

5. The weight on the drivers of a locomotive is 183,900 lb. and the tractive force with the mean effective pressure at $\frac{4}{5}$ of the boiler pressure is 30,700 lb. What is the ratio of the weight on the drivers to the tractive force?

26. Proportion.—A proportion shows the equality of two ratios. For instance, the ratio of 8 to 4 is equal to the ratio of 12 to 6, since each of these ratios is 2. This might be written

$$8 : 4 :: 12 : 6$$

and would be read, 8 is to 4 as 12 is to 6, or the ratio of 8 to 4 is equal to the ratio of 12 to 6. In this proportion 8 is called the first term, 4 the second, 12 the third, and 6 the fourth term. The two outside or the first and the last terms are called the *extremes* and the two inside or the second and the third terms are called the *means*.

An important rule to remember in proportion is that the product of the means is equal to the product of the extremes, for example in the proportion

extremes
↓ ↓
8 : 4 : : 12 : 6
↑ ↑
means

Product of extremes	=	Product of means
8×6	=	4×12
1st 4th		2nd 3rd
48	=	48

or in other words, the first term multiplied by the fourth term equals the second term multiplied by the third term.

This rule will enable us to find any one of the four terms if the other three of them are known, since either mean may be found by dividing the product of the extremes by the other mean. The same will hold true to find either extreme. Proportion gives us a convenient method for finding change gears for a lathe. For example, if we wish to cut 12 threads per in. with a lead screw having 4 threads per in., in this case, while the tool carriage advances 1 in., the stock must make 12 turns. The lead screw having 4 threads per in. will feed the carriage 1 in. in making 4 turns, hence the lead screw must turn 4 times while the stock turns 12 times. If we have a gear with 24 teeth on the stud and wish to find the number of teeth required on the gear to be placed on the lead screw, we may write the proportion thus:

$$4 : 12 :: 24 : x$$

where x is written in place of the number of teeth on the change gear. In this case

$$x = \frac{\overset{3}{\cancel{12}} \times 24}{\cancel{4}} = 72 \text{ teeth}$$

The diameters of pulleys and gears are indirectly proportional to their speeds in r.p.m. or $D : d :: n : N$, in which

D = diameter of driver
d = diameter of driven
N = speed of driver
n = speed of driven.
r.p.m. = revolutions per minute.

PROBLEMS

6. (*a*) If the diameter of a pulley is 36 in., and makes 120 r.p.m. what is the r.p.m. of a second pulley belted to the first if its diameter is 16 in.?

(*b*) Two pulleys have diameters of 32 and 25 in. If the first makes 120 r.p.m., what is the r.p.m. of the other?

7. Find the diameter of a pulley that is to make 90 r.p.m., if it is belt connected to a pulley which makes 150 r.p.m. and has a diameter of 12 in.

8. What will be the pitch diameter of a gear that is to make 40 r.p.m., if it is to mesh with a gear 24 in. in diam., which makes 42 r.p.m.?

9. In a mixture of copper, lead, and tin there are 4 parts copper, 3 parts lead, and 1 part tin, how many pounds of each would there be in 176 lb. of the mixture?

Here the total number of parts would be $4 + 3 + 1$ or 8, hence we have the following:

For the copper	$4 : 8 :: x : 176$
For the lead	$3 : 8 :: x : 176$
For the tin	$1 : 8 :: x : 176$

10. A solder is made of 5 parts zinc, 2 parts tin, and 1 part lead. How many parts of each metal in a mixture of 72 lb. of the solder?

11. The formula for making lining metal for bearings calls for 87 parts lead and 13 parts antimony. How many pounds of each metal in a mixture of 375 lb?

12. A machinist gets $3 a day and a helper gets $1.50 per day. How much would the helper receive when the machinist gets $75 providing both work the same number of days?

13. A planer has a cutting speed of 32 ft. per min. and a return speed of 150 ft. per min., what is the ratio of the cutting speed to the return speed?

14. The cutting speed on a 36-in. planer is set at 45 ft. per min. The ratio of the return speed to the cutting speed is 2.9. How long would it take to plane a surface 18 in. wide and 20 ft. long with a feed of ¼ in. per cut and allowing an over-travel of 6 in. at each end of the stock?

15. A freight locomotive consumes 20 gal. of fuel oil per mile. How many pounds of oil will be consumed on a 120-mile run if the oil weighs 7.8 lb. per gal.?

16. If a steam pump delivers 2.8 gal. of water at each stroke and makes 48 strokes per min., find how long it will take to pump 2000 gal.

17. If the construction cost of 100 miles of railway is $4,500,000, what is the cost of 286 miles at the same rate?

18. The weight of 10 ft. of ¼-in. iron pipe is 4.2 lb. What is the weight of 480 ft. of the same pipe?

19. A train is made up as follows:

Engine and tender	weighing 380,000 lb.
Mail car	weighing 102,000 lb.
Baggage car	weighing 110,000 lb.
Smoking car	weighing 115,000 lb.
Pullman sleeper	weighing 120,000 lb.
Pullman sleeper	weighing 118,400 lb.
Diner	weighing 130,400 lb.

What proportional part of the weight of the whole train is that of the engine and tender? What part of the weight of the whole train is taken up by the diner?

20. If a steel rail weighs 100 lb. per yd. of length, what is the weight of 40 ft. of this size rail?

21. A train travels 420 miles in 12 hr. How far will it travel at the same rate in 20 hr. and 30 min.?

22. A railroad company placed an order for 50,000 tons of rail for $1,400,000. At the same rate what would 65,000 tons cost?

23. An order for 16 castings requires 1240 lb. of steel. How much metal is required for 24 castings of the same size and pattern?

24. An apprentice received in 30 days $44.60 pay. The next month he worked 28 days at the same rate. How much did he receive?

25. Find the weight of 2460 ft. of insulated copper wire if 1000 ft. weigh 116 lb.

26. The ratio of weight of the insulated wire in Prob. 25 to the wire when bare is 116:50. What is the weight of the wire if bare?

Calculation of Grades.—In calculating the grade of a roadbed, the rise in 100 ft. horizontal distance is found.

For example, if a railroad track rises $1\frac{1}{2}$ ft. for every 100 ft. horizontal distance, the grade is $\frac{1\frac{1}{2}}{100}$ or $1\frac{1}{2}$ per cent.

Example.—On the main line of the Pennsylvania Railroad between Altoona, Penna., and Gallitzin, Penna., the roadbed rises approximately 90 ft. to the mile. The grade is therefore $^{90}/_{52.8}$ or 1.70 per cent. (52.8 is the number of 100 ft.-lengths in 1 mile.)

27. A roadbed rises 2.75 ft. in 400 ft. horizontal distance. What is the grade? What is the rise per mile distance?

28. A cog railway up the side of a mountain has at parts of the route a grade of 20 per cent. What rise does this represent for every mile of horizontal distance?

Proportion for Similar Figures.—In Fig. 22 are shown *similar* triangles, that is, three-sided figures

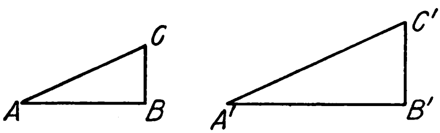


FIG. 22.

whose angles A and A^1 are equal and whose respective sides have the same direction or are parallel. In these figures these proportions are true:

$$\frac{AB}{A'B'} = \frac{BC}{B'C'} \quad \text{also} \quad \frac{AB}{BC} = \frac{A'B'}{B'C'}$$

In other words the ratio of any two sides on either triangle equals the ratio of similar sides taken in the same order on the other triangle.

To Measure Heights and Distances by Proportion.—As shown in Fig. 23, the height CD of the telegraph pole may be found by sighting from the ground across the top of a pole AB (of known length placed a known distance BD from the bottom of the pole) to the top of the pole C.

The similar triangles EBA and EDC are formed and the following proportion is true:

$$\frac{CD}{AB} = \frac{DE}{BE}$$

from which

$$CD = AB \times \frac{DE}{BE} = \text{height of pole.}$$

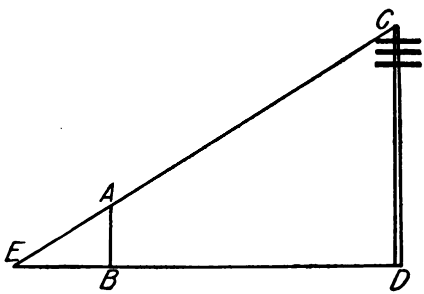


FIG. 23.

The distance BE is measured and knowing distances DBE and BE, distance CD can be easily calculated.

Example.—The top of a telegraph pole, Fig. 24, is sighted across a 6-ft. pole placed 70 ft. from the foot of the telegraph pole, the observer sighting from the ground at a distance of 10 ft. from the foot of the 6-ft. pole. Find the height of the telegraph pole.

$$\text{ht.} = \text{distance } CD = AB \times \frac{DE}{BE} = \frac{6 \times 80}{10} \text{ or } 48 \text{ ft.}$$

Another method may be used employing two poles, as follows: From the end of a short pole, Fig. 25, one observer sights the top and bottom of the telegraph pole and a sec-

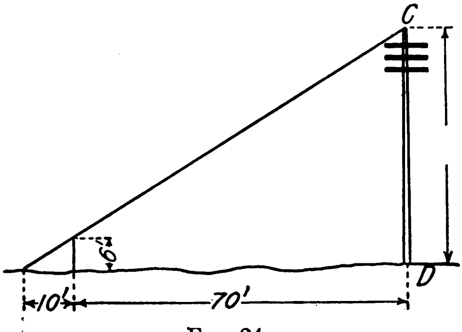


FIG. 24.

ond observer notes where the line of vision crosses a second and longer pole. From his reading the distance

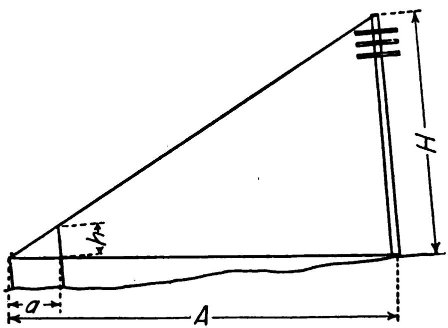


FIG. 25.

"h" may be found. The distances from the top of the short pole to the foot of the telegraph pole, that is distance "A" and also distance "a" (from the top of the short pole to where the tape touches the longer pole), are measured. From these measurements we can find "H" as follows:

$$\frac{H}{h} = \frac{A}{a} \text{ and } H = h \times \frac{A}{a}$$

Example.—In using two poles to measure a telegraph pole height the distance between the lines of vision on the longer pole was 7 ft. The top of the short pole was 76 ft. from the foot of the telegraph pole and the tape touched the longer pole at the 8-ft. mark, the end of the tape being held on the top of the shorter pole. Therefore since

$$H = h \times \frac{A}{a}$$

we have

$$H = \frac{7 \times 76}{8}$$ or 66.5 ft., the height of the telegraph pole.

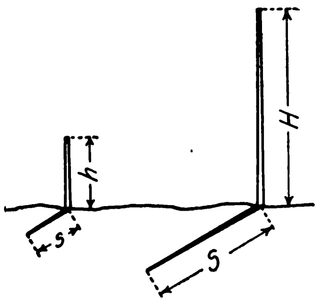


FIG. 26.

If the sun is shining the height of an object may be found by measuring the length of its shadow and the length of shadow "s" of a pole of known length. Then according to Fig. 26 we have the rule

$$\frac{H}{h} = \frac{S}{s} \text{ and } H = h \times \frac{S}{s}$$

Example.—In measuring the height of a tower whose shadow is 48 ft. long it is found that at the same time a pole 12 ft. long casts a shadow 4 ft. long. How high is the tower?

Solution.—

$$H = h \times \frac{S}{s}$$

or

$$H = \frac{12 \times 48}{4} = 144 \text{ ft., height of tower.}$$

PROBLEMS

29. What is the height of a signal pole whose shadow is 15 ft. when a 10-ft. pole at the same time casts a shadow of 2 ft. 6 in.?

30. The length of shadow of a telegraph pole to the first cross arm is 22 ft. An 8-ft. pole at the same time casts a shadow 3 ft. and 4 in. long. What is the distance from the ground to the first cross arm?

CHAPTER VI

MEASUREMENT OF ANGLES

27. Definitions and Problems.—A point has neither length, breadth, nor thickness but position only.

A straight line is one that does not change its direction.

A curved line is one that changes its direction at every point. Both straight and curved lines are measured in *simple units of length as inches, feet, yards, miles, etc.*

When two straight lines cross or *intersect* as at *O* in Fig. 27, an angle is formed. This is read angle *AOB*, the letter at the *vertex O*

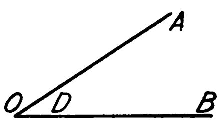


FIG. 27.

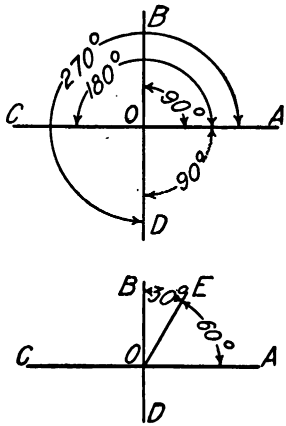


FIG. 28.

being read second. The size of the angle (abbreviated ∠) depends *only* on the amount *of opening* between the sides, *AO* and *BO*, and *not* on the *length* of the sides. The angle may also be indicated by a letter at the vertex, as the angle *D* for example.

Angles are measured in degrees (°), minutes (′), and seconds (″). The entire angle about a point, Fig. 28, represents 360 degrees (abbreviated °). This number is chosen as it is easily divisible by the prime numbers 1, 2, 3, etc., without a remainder.

If, as in Fig. 28, we take ¼ the entire angle about the point *O*, we have the angle *AOB* which equals ¼ of 360° or 90° and is called a *right angle.* The sides *AO* and *BO* of the angle are said to be *perpendicular* to each other since they form a right angle. In the figure the angle *AOC* = 180° and the angle *AOD* measured *counter-clockwise* is 270°. The angle *AOD* measured *clockwise* (as indicated by the dotted arrow) equals 90°. Angle *AOE* in the lower figure is equal to ⅔ of a right angle and is ⅔ of 90° or 60°, and angle *BOE* = 30°.

Every degree contains 60 min. (′) and every minute contains 60 sec. (″). One-seventh of a right angle = $\frac{1}{7}$ of 90° or $12\frac{6}{7}$° or 12° $51\frac{3}{7}$′ or 12° 51′ 26″.

A *horizontal line* is one which has the same direction as the horizon, as line *AC*, Fig. 28.

A *vertical line* is one perpendicular to a horizontal line as *BD*, Fig. 28.

To *bisect* an angle or a line means to divide it into two equal parts. To *trisect* means to divide it into three equal parts.

An angle not a right angle is called an *oblique angle.*

An *acute* angle is one less than a right angle and an *obtuse* angle is one greater than a right angle.

PROBLEMS

1. If two straight lines intersect and one of the angles is a right angle, what kind of angles are the other three? Show by means of a figure.

2. In Fig. 29, if the two straight lines *AB* and *CD* cross at *O* what relation have the angles *AOC* and *BOD*? Are the angles *AOD* and *BOC* (called *vertical angles*) equal? Are the angles *AOD* and *BOD* equal? How many degrees are there in the two angles *AOD* and *BOD* added together?

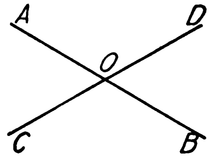


FIG. 29.

3. Find the sum of the three following angles: 26° 13′ 15″; 89° 27′; 45° 30′ 18″.

4. Do you think it is possible to draw more than one perpendicular to a given line at a given point? Give your answer clearly in complete sentences.

5. What size angle is obtained by bisecting an angle of 22½°; also by trisecting it? What part of a right angle would each of these angles then be?

6. Two angles are said to be "complementary" when their sum is equal to a right angle, and each angle is called the "complement" of the other. What is the "complement" of each of the following angles: 22° 15′; 45° 30′ 16″; 75° 32′ 30″; 72° 15′; 30¼°?

7. Two angles are said to be "supplementary" when their sum is equal to two right angles, and each angle is called the "supplement" of the other. What is the "supplement" of each of the following angles: 120° 30′ 15″; 133° 20′; 72° 15′ 45″; 105°?

8. What angle does a plumb line make with a horizontal line?

9. Find the sum of the following angles: 27° 30′ 40″; 89° 50′ 18″; 54¼°; 8° 7′ 19″; 45° 38′ 15″.

10. Find the sixth of an angle of 45°. What size angle is obtained by bisecting an angle of 35° 30′; also by trisecting it? What relation would each of the angles then bear to a right angle?

11. A wheel has twelve spokes. What angle is formed by the center line of two adjacent spokes?

12. There are 21 angles about a point *a*. What is the value of each?

13. Find the sum of the following angles: 46° 18′; 27° 30′ 15″; 82° 45′ 45″.

14. What part of a right angle are the following: 11° 15′; 18°; 30° 30′; 42° 15′?

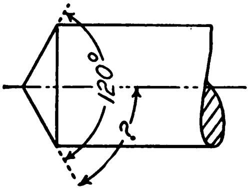


Fig. 30.

15. In facing a piece of work in a lathe as in Fig. 30, at what angle must the slide be set off in order to obtain a total angle of 120° for the work?

CHAPTER VII

MENSURATION OF RECTANGLES AND OTHER FOUR-SIDED FIGURES

28. Definition.—Mensuration of surfaces means the measurement of surfaces. Thus far, all our measurements have been those of length only. We have used the ordinary scale having 12 in. per ft.

Larger measurements of length may be made in feet,

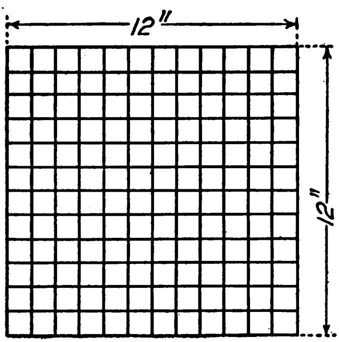


FIG. 31.

yards, rods, or miles. Small measurements are made in inches or fractions of an inch.

In measuring *areas* or surfaces, we can no longer use the inch, the foot, etc., but we must use the *square inch*, the *square foot*, the *square yard*, etc. A square inch is a surface 1 in. long and 1 in. wide. Thus we see that in measuring surfaces we measure in *two* directions.

A square foot is a surface 1 ft. long and 1 ft. wide. If we divide a square foot, Fig. 31, both ways by drawing lines 1 in. apart we divide it into 144 parts each of which is a square inch. We see that we have 12 sq. in. in one horizontal row and that there are 12 such rows. Hence we have in 1 sq. ft., 12 × 12 or 144 sq. in. If then we wish to reduce square feet to square inches we must multiply the number of square feet by 144 and the product will be square inches.

If we wish to reduce square inches to square feet we divide the number of square inches by 144 and the quotient will be square feet. The following table is the one used in square measure:

144	sq. in.	= 1 square foot (sq. ft.)
9	sq. ft.	= 1 square yard (sq. yd.)
272¼	sq. ft. or 30¼ sq. yd.	= 1 square rod (sq. rd.)
160	sq. rd.	= 1 acre (A)
640	acres	= 1 square mile (sq. mi.)

29. The Rectangle.—The simplest figure that we deal with is one that has four sides and four *square corners* or *right angles*. Such a figure is called a *rectangle*.

A *square* is a rectangle whose four sides are equal. Ordinarily we speak of a rectangle as being a figure whose length is greater than its width. In finding the area of a rectangle we can divide it into square inches by drawing lines both ways 1 in. apart. The area may be found by finding the number of "little" squares contained in the figure.

Thus, Fig. 32 shows a rectangle 7 in. long and 5 in. wide divided into square inches. We see that there are 7 sq. in. in 1 row and that there are 5 rows, hence the number of square inches in the figure is 5 × 7 or 35 sq.

in. We thus see that the area may be found by multiplying the *length* by the *width*. Thus, the area of a rectangular window 7 ft. long and 4 ft. wide is 7×4 or 28 sq. ft. Or if we let L equal the length and W equal the width, then the area equals $L \times W$.

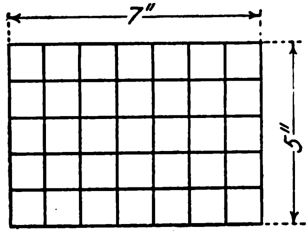


FIG. 32.

In the following problems find the area by multiplying the length by the width.

NOTE.—*Care must be taken to use the same units for the length and width. Thus, you cannot multiply inches by feet and call the product square inches or square feet. Both length and width must be in inches or both in feet.*

PROBLEMS

1. A machine shop is 150 ft. long and 75 ft. wide. How many square feet of floor space does it contain?

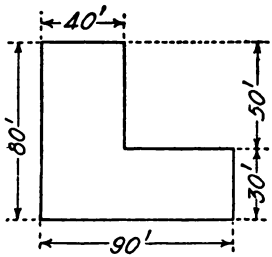


FIG. 33.

2. How many square feet of surface in the four sides of the shop mentioned in Prob. 1, if the sides are 12 ft. high? (Draw a sketch of the sides and ends and put in the dimensions.)

3. A machine shop in the shape of an "*ell*" has the dimensions as shown in Fig. 33. How many square feet does it contain?

(Divide it into two rectangles, find the area of each separately, and add together.)

4. Find the area of a square piece of steel plate measuring 6 ft. on a side. What is its weight at 10 lb. per sq. ft. of surface?

5. Find the number of square inches in a steel strip 7 ft. long and 18 in. wide.

6. The cylindrical part of an air drum measures 4½ ft. by 64 in. before bending. How many square inches does it contain?

7. The grates in a locomotive are 110 in. long and 6 ft. wide. How many *square feet* of grate surface are there in this type of locomotive?

8. A locomotive-erecting shop is 404 ft. long and 94 ft. wide. It has one floor. Find its floor area in sq. ft.

30. Finding the Length and Width.—In many cases we know the area and one of the dimensions and wish to find the other dimension. For example, in Fig. 34, how long must a rectangle be that is 5 in. wide in order to contain 35 sq. in.?

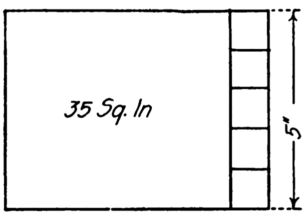


FIG. 34.

Here we have 5 sq. in. in each strip, across the width for every inch of length; hence to contain 35 sq. in. the length must be 35 ÷ 5 or 7 in. *Hence to find the length when we know the area and the width we divide the area by the width and the quotient is the length. To find the width when we know the area and length, divide the area by the length and the quotient is the width.*

If a = the area

l = the length

w = the width

then $$l = \frac{a}{w} \text{ and } w = \frac{a}{l}$$

9. Fill out the following blanks for dimensions and areas of locomotive grates:

Type	Length, in.	Width, in.	Area, sq. ft.
Passenger	111	....	61.86
Passenger	110¾	....	55.4
Freight	110¼	....	55.13
Freight	120	40	....

10. The plans for a machine shop call for a length of 125 ft. and a width of 75 ft. It is decided to increase the floor area by 625 sq. ft. How much must be added to the length if the width is to be increased 5 ft.?

11. In order to keep the velocity of the steam entering a cylinder from becoming too high it is found that the area of the steam port must be 21⅝ sq. in. If the width is 1¼ in., how long must it be made?

31. The Square.—It has already been stated that a square is a rectangle whose four sides are equal. If we wish to find the area of a square we multiply the length of the side by itself, or in other words we "square" the side. Thus the area of a square 8 in. on a side is equal to 8×8 or 64 sq. in. Multiplying a number by itself is therefore called "squaring" it. This operation is often indicated by writing a small figure 2 at the right of and a little above the number as 8 squared is written "8^2," meaning that 8 is to be multiplied by itself, and in this case 8^2 would equal 8×8 or 64 and would be read "8 squared."

PROBLEMS

12. Find the value of 16 squared.
Find the value of 13 squared.
Find the value of 15^2.
Find the value of 275^2.

13. Find the area of a square 18 in. on a side.

14. Find the area of a square steel plate measuring 19 in. on a side.

15. (*a*) A machine shop is 81 ft. square. How many square feet of floor space are there in the shop? (*b*) If there are 25 machines in the shop, find the average number of square feet of floor space per machine.

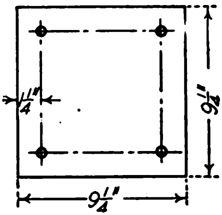


FIG. 35.

16. (*a*) Find the area of the square boiler patch shown in Fig. 35. (*b*) If the boiler pressure is 200 lb. per sq. in., what would be the total pressure against the patch? (*c*) If the rivets are placed 1¼ in. from the edge and spaced 2¼ in. between centers, how many rivets are required for the outside row of rivets around the patch?

If we wish to find the side of a square having a given area we must perform the opposite operation to "squaring," that is, we must find a number such that if we multiply this number by itself or square it, we will get the given area. This operation is called finding the "square root" and is indicated by the sign ($\sqrt{\ }$) known as the square root sign. Thus, the length of the side of a square containing 16 sq. ft. is equal to the square root of 16 ($\sqrt{16}$) or 4, because $4 \times 4 = 16$.

PROBLEM

17. Find the side of each of the squares having an area of 9, 36, 25, 16, 81, and 100 sq. ft. respectively.

In this problem it is easy to find out what number multiplied by itself will give the given area. In case we wish to find the side of a square having a large area, as for instance 1369 sq. ft., we cannot tell off hand what the length of the side is and we must find the "square root" in the following manner:

32. Finding the Square Root.—Begin at the right or decimal point and divide the number into groups of two figures each by drawing a line between the 2nd and 3rd, the 4th and 5th, 6th and 7th figure thus, 13′69. Find the largest number which when squared will go into the first left-hand group, 13 in this case. This number is the first part of the root or the first figure in the answer and is 3 in this case, since 4 squared or 4 × 4 is 16 which is too large. We must now square the "3," which gives us 9, and arrange the work similar to that of division as here shown. Subtract the square of

$$\begin{array}{rl} & 13'69(\underline{3}\\ 3^2 = 3 \times 3 = & \underline{9}\\ & 4 \end{array}$$

the number found or 9 from the 1st group or 13 which leaves 4 as a remainder. The next step is to bring down the following group, 69 in this case, making our new number 469.

$$\begin{array}{rl} & 13'69(\underline{3}\\ 3^2 = 3 \times 3 = & \underline{9}\\ 3 \times 2 = 6) & 4\ 69\\ \text{Trial divisor.} & \end{array}$$

Next double the figure in the answer or root already found, which in our problen is 3 × 2 or 6 and use this as a trial divisor. Now find how many times the trial divisor is contained in the remainder or 469 by leaving off the first figure to the right or the 9, which in this case would be 46 ÷ 6 or 7. Place this number found in the answer and also to the right of the trial divisor which would give us 37 in the answer and 67 as a final divisor.

```
                          13'69 (37 Ans.
        3² = 3 × 3 =       9
                         ) 469
3 × 2 = (67) × 7 =      ) 469
   Final divisor.
```

Next multiply the final divisor, 67, by the last figure found in the answer or 7 and subtract the product from 469. If we wish to prove our answer and see if it is correct we can "square" it and get the number we started with, thus 37 × 37 is 1369.

The square root of decimals and of whole numbers and decimals is found the same as in the case of whole numbers except that we point off the groups by beginning at the decimal point and advancing toward the right for the decimal. For example: If we want to find the square root of 35378.345 we point off as follows:

3'53'78.34'50

Since there are only three figures in the decimal we must add a cipher in order that there will be two figures in each group marked off. The square root is found as follows:

```
                              3'53'78. 34'50 (188.09+
                 1 × 1 =      1
                             )253
      2 × 1 = 28 × 8 =       )224
                             ) 2978
   2 × 18 = 368 × 8 =        ) 2944
                             )    343450
2 × 188 = 37609 × 9 =        )    338481
```

In the above case the answer has five figures since there are five periods or groups in the given number. As there are two *groups* to the right of the decimal point in the number, there must be *two places* to the right of the decimal point in the answer.

PROBLEMS

18. Find the square root of 1296, 16,769, 494,209, 18,090.25.

19. A square pipe has an area of 150.0625 sq. in. What is the length of its side?

20. A fan outlet 4 ft. 6 in. wide and 3 ft. 4 in. high is to be branched off into two equal square pipes. What must be the length of the side of each pipe if the combined area of both pipes is to equal the area of the fan outlet?

21. Find the square root of 2 to three decimal places. Find the square root of 3 to three decimal places.

22. A rectangle has an area of 1728 sq. ft. If the length is three times the width find the length and width.

(NOTE.—If the length is three times the width it can be divided into three equal squares. See Fig. 36.)

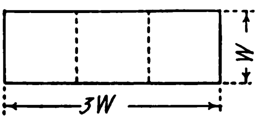


FIG. 36.

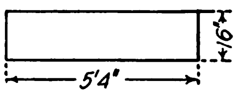


FIG. 37.

23. A fan outlet, Fig. 37, on a hot blast heating system is 5 ft. 4 in. wide and 16 in. high. The pipe leading from the fan is to be square but must have the same area as the fan outlet. What must be the size of the square pipe?

Which would be the cheaper, to build the square pipe or one having the same dimensions as the fan outlet? Tell why.

33. Other Four-sided Figures.—Sometimes we have to deal with four-sided figures whose angles are not right angles but whose opposite sides are equal and parallel, that is, run in the same direction. Such a figure is called a *parallelogram*. The area of a parallelogram is found by multiplying the length of one of the parallel sides or base by the width or distance measured squarely across the figure as shown in Fig. 38. In the case of the rectangle which we had first taken up, the width was equal to the short side. In the case of a parallelogram the width is *not* equal to the length of the side but must be measured *squarely* across from one side to the other.

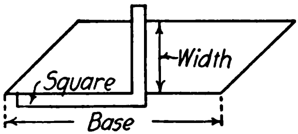


FIG. 38.

PROBLEM

24. Find the area of each of the following size boiler patches having the shape as shown in Fig. 39 and dimensions as given in the table:

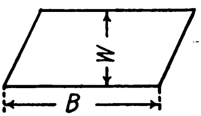


FIG. 39.

B	*W*	Area
2 ft. 3 in.	8 in.	
13 in.	7 in.	
3 ft. 6 in.	9 in.	

34. The Trapezoid.—Sometimes a four-sided figure has only two of its sides parallel, as in the figure shown. Such a figure is called a *trapezoid*. The area of a trapezoid may be found by multiplying half the sum of the parallel sides by the distance between the two sides. The area of Fig. 40 is ½ of (8 + 12) or 10 times 6 or 60 sq. in., or if *a* and *b* equal the length of the parallel sides or bases and *c* equals the distance between them the

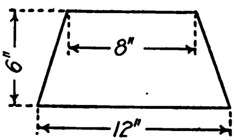


FIG. 40.

$$\text{Area} = \frac{a + b}{2} \times c$$

25. Find the area of boiler plates having the shape shown in Fig. 41 and dimensions as given in the table:

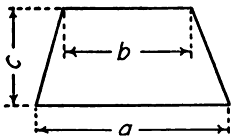


FIG. 41.

a	*b*	*c*	Area in sq. in.
6 ft.	62 in.	102 in.	
6 ft. 9 in.	77 in.	83 in.	
6 ft. 10 in.	54½ in.	65½ in.	
11 ft. 10 in.	101 in.	70 in.	
8 ft. 5 in.	79½ in.	93 in.	
98½ in.	90½ in.	101½ in.	

CHAPTER VIII

MENSURATION OF TRIANGLES

35. The Triangle.—A figure having three sides is called a triangle. Fig. 42 shows a triangle having a base of 21 in. and an altitude or height of 12 in. The altitude is always taken as the distance measured "squarely" from the vertex to the base as shown in the figure. The area of a triangle is equal to ½ the product of the base and the altitude, or if

$$a = \text{the altitude}$$

and $$b = \text{the base}$$

then the area "A" $= \frac{a \times b}{2}$ or $A = \frac{ab}{2}$

Thus the area of the triangle in Fig. 42 is

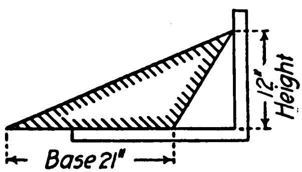


FIG. 42.

$$\frac{21 \times \overset{6}{\cancel{12}}}{\cancel{2}} = 126 \text{ sq. in.}$$

PROBLEMS

1. Find the area of the following triangles:

Base, b, in.	Alt., a, in.	Area
16	24	
32	17	
45	63	
25	63	

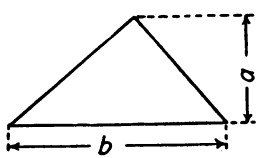


FIG. 43.

2. The tip of the roof of a machine shop is 9 ft. above the sides, as per Fig. 44. If the shop is 36 ft. wide, how many square feet of siding are required to cover both gable ends?

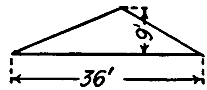


Fig. 44.

3. Find the area of the fire-box side sheet in Fig. 45, by finding the area of each triangle and adding together.

4. Find the area of crown and side sheets in Fig. 46 by finding the area of a rectangular piece required from which the sheet may be cut to the size given and subtract the combined areas of the triangles cut off.

5. The area of a triangle is equal to 75 sq. in. and the base is 25 in. What is the altitude?

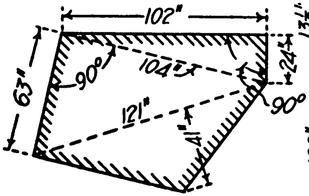


Fig. 45.

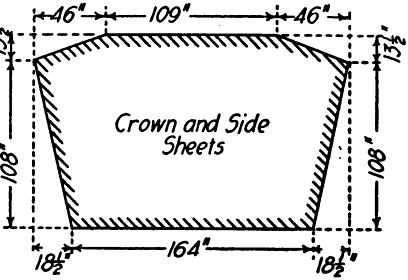


Fig. 46.

Note.—Since the area equals ½ the base times the altitude, the altitude must equal twice the area divided by the base, or the altitude,

$$a = \frac{2 \times A}{b}$$

and likewise the base, or

$$b = \frac{2 \times A}{a}$$

6. The altitude of a triangle is 165 in. and the area is 6600 sq. in. What is the base?

36. The Right Triangle.—When one angle of a triangle is equal to 90° or, in other words, when one side is square to another, we have a *right triangle* as in Fig. 47.

The longest side of a right triangle is called the "hypothenuse" and the two short sides, or the two enclosing the right angle, are called the "legs" of the right triangle. A definite relation exists between the three sides of a right triangle. If we draw a square on each side, as in Fig. 48, the sum of the areas of the squares on the two short sides is equal to the area of the square on the long side or hypothenuse. As in Fig. 48, the squares on the small sides contain $3^2 + 4^2 = 9 + 16$ or 25 sq. in. The long side is 5 in. and the number of square inches in the square drawn on this is 5×5 or 25 sq. in., which is the same as the sum of the squares on the two short sides.

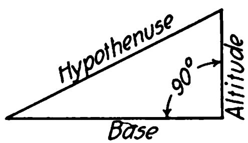


FIG. 47.

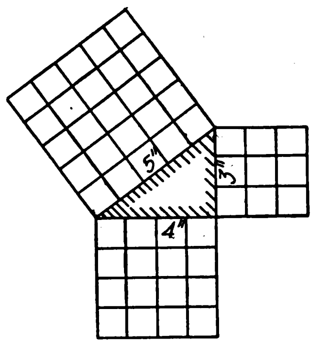


FIG. 48.

This relation enables us to find the third side if we know any two of the sides. The following will be of aid in this work.

To find the hypothenuse or long side of a right triangle *square each of the two short sides, add these squares together and then extract the square root of their sum.*

If a and b represent the two short sides of a right triangle, Fig. 49, and c the hypothenuse, this rule may be written:

$$c = \sqrt{a^2 + b^2}$$

To find either short side when the hypothenuse and the other side are known *subtract from the square of the hypothenuse* the square of the *known short side and extract the square root of the difference.*

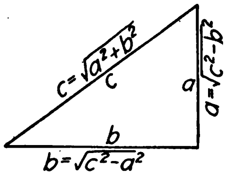


Fig. 49.

Using the same notation for the sides of a right triangle as is used above, this rule may be written as follows:

$$a = \sqrt{c^2 - b^2} \text{ and } b = \sqrt{c^2 - a^2}$$

PROBLEMS

7. As shown in Fig. 50, the side of a 3-in. square nut is $4\frac{5}{8}$ in. What is the distance across corners?

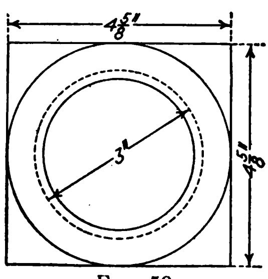


Fig. 50.

8. Find the missing sides in the following triangles:

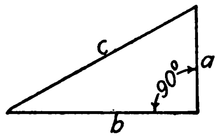


FIG. 51.

a	b	c
42 in.	25 in.	....
60 in.	....	72 in.
....	96 in.	144 in.

9. Assuming the center line of the drivers to be on the center line of the cylinder, Fig. 52, find the distance (x) from the center

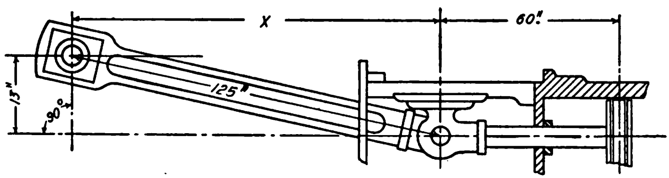


FIG. 52.

of the driver to the center of the crosshead pin when the crank pin is on the top quarter. Is the piston on the middle of its stroke when the crank pin is on the top quarter? If not, on which side of the middle of its stroke is it? How much is it back of the middle?

10. Figure 53 shows the center lines of a roof truss. Find the missing dimensions A, B, C, and D. (NOTE: Draw each triangle separately and find missing side.)

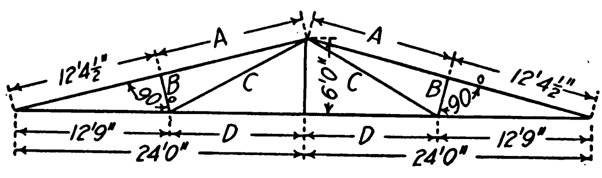


FIG. 53.

11. It is desired to shape a 2¾-in. square nut from a round piece. What must be the diameter of the round bar if the 2¾-in. square nut measures 4¼ in. on a side?

12. A 3¾-in. square nut, Fig. 55, measures 8¼ in. across corners. What is the length of its side?

13. Find the height of an equilateral (equal sided) triangle, Fig. 56, 1 ft. on a side.

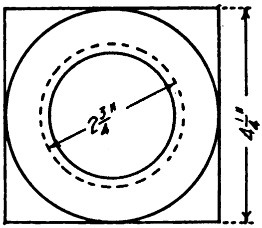


FIG. 54.

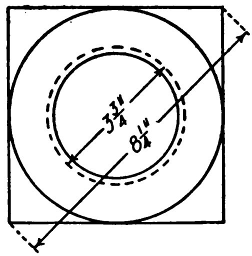


FIG. 55.

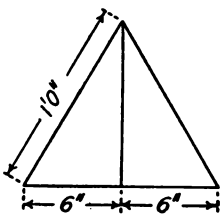


FIG. 56.

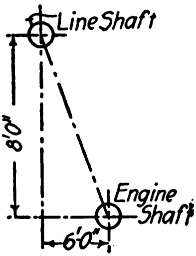


FIG. 57.

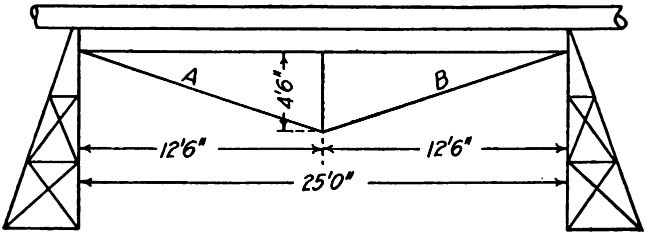


FIG. 58.

NOTE.—Divide it into two equal right triangles by drawing a line from the vertex to the base as in the figure.

14. A line shaft is 8 ft. above and 6 ft. to the left of an engine shaft. Find the distance between centers of the two shafts.

15. Figure 58 shows the structural work supporting a steam pipe across a passageway. Find the length of the members *A* and *B*.

MISCELLANEOUS PROBLEMS FOR PRACTICE

16. What is the area of a parallelogram whose base is 14 ft. and height or altitude is 9 ft.?

17. What is the height of a parallelogram whose base is 24 in. and whose area is 104 sq. in.?

18. A rectangular piece of land 20 yd. wide is to be increased in area 600 sq. ft. How much will the length of the rectangle be increased if the addition is 20 yd. wide?

19. At 85 cts. per sq. yd. what would be the cost of laying a foundation 20 ft. long and 10 ft. 6 in. wide?

20. What is the length in in. of the side of a square sheet of metal containing 2 sq. yd.?

21. If to obtain the volume or contents of a regularly shaped object we multiply the length by the width and by the thickness, what would be the volume in cubic inches of a ¼-in. steel plate 3 × 4 ft.?

22. What is the weight in lb. of the above plate if steel weighs .28 lb. per cu. in. and we allow for three holes each 2 in. square?

23. How many square inches will have to be removed from a ⅜-in. steel plate in order to reduce its weight 8 lb.?

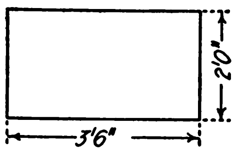


FIG. 59.

24. Figure 59 shows a steel plate 3/16 in. thick. What is the figure called, and why? What is the area of the plate in square inches and its volume in cubic inches? Calculate its weight at .28 lb. per cu. in.

25. Of two lots of ground each containing 160,000 sq. ft., one in the shape of a rectangle 200 ft. long, or one in the shape of a square, which takes the more fence to go around it?

26. What is the cost of enclosing a rectangular field 20 rd. long and containing 120 sq. rd. if the cost of fencing is 12 cts. per ft.?

27. A 1/4-in. boiler sheet is 8 ft. long and 4 ft. 6 in. wide. What will be its weight after deducting 9 lb. for rivet holes?

28. How many sheets of drawing paper 9 in. × 12 in. can be cut from a roll 3 ft. wide and 15 yd. long, if no allowance is made for waste? How many sheets of the same size could be cut from the same roll if 1/2 in. is allowed on each side for cutting?

29. At 40 cts. per sq. yd. what will be the cost of plastering the sides and ceiling of a room 10 ft. wide, 14 ft. long and 8 ft. high?

30. It is desired to add 8000 sq. ft. to the floor area of a storehouse 60 ft. wide. If the addition is also 60 ft. wide, what will be the length of the addition?

31. Find the distance across the corners of the following sized squares: 1 in., 1.5 in., 2 in., and 3 in. on a side.

32. A triangular-shaped steel plate, as in Fig. 60, has a base of 6 in. and a height of 4 in. How many square inches of surface does it contain? If the plate is 3/8 in. thick what is its weight?

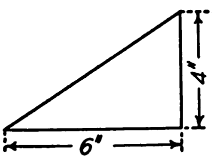


Fig. 60.

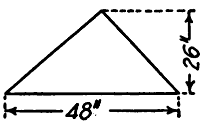


Fig. 61.

33. Figure 61 shows the form of a steel plate 1/4 in. thick. What is the form of the plate? What is its base, its height, and area in sq. in.? Find the volume of the plate in cu. in. and its weight in lb.

34. Find the distance between the centers of the pulleys shown in Fig. 62.

35. Figure 63 shows the center lines of a roof truss. Find the lengths of members A, B, and C.

36. Which is the larger, a parallelogram with base 7 in. and height 7 3/8 in. or a triangle with base 7 in. and height 2 5/8 in.? What is the difference in area of these figures?

37. How many more feet of fencing will it require to enclose 10,000 sq. ft. in the form of a right triangle with base of 100 ft.

than in the form of a square? If the fencing costs 16 cts. per ft. of length what will be the difference in cost in the two cases?

38. What is the ratio in area of a triangle whose base is 6 in. and height $4\frac{1}{2}$ in. and one whose base is 12 in. and height $5\frac{3}{4}$ in.?

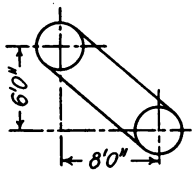


FIG. 62.

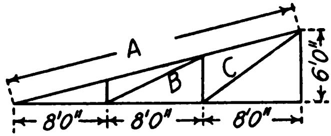


FIG. 63.

39. Figure 64 shows the dimensions for a storehouse. How many yards of roofing 30 in. wide will it take to cover the roof if the roofing laps 3 in.?

40. A passenger train and a freight train start from a station at the same time. The passenger train travels north at the rate of 40 miles per hr., the freight east at the rate of 25 miles per hr. How far apart are the trains at the end of 3 hr.?

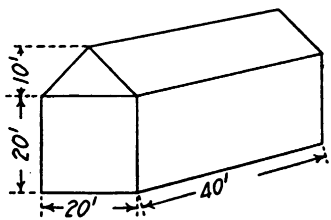


FIG. 64.

41. How long a ladder is required to reach to a window 15 ft. from the ground if the foot of the ladder is placed 10 ft. from the side of the building?

42. If the patch on a boiler is of the shape shown in Fig. 65, how many square inches does it contain and what is the total outward pressure acting on it with a boiler pressure of 210 lb. per sq. in.?

43. How many square feet in a board 12 ft. long if it is 8 in. wide at one end and 10 in. wide at the other?

44. Figure 66 shows a steel plate $\frac{3}{16}$ in. thick. What is the shape of this figure, its area in sq. in., the volume of the plate in cu. in., and its weight at .28 lb. per cu. in.?

45. What is the ratio of the areas of a trapezoid 6 in. high with upper and lower bases of 10 in. and 12 in. respectively, and a parallelogram 8 in. high and $9\frac{1}{2}$ in. base?

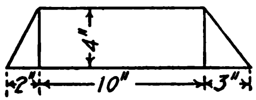


Fig. 65.

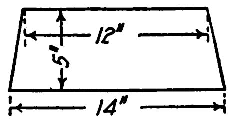


Fig. 66.

46. Find the area of an equilateral triangle 1 in. on each side.

47. Find the area of a regular hexagon 1 in. on a side by dividing it into six equilateral triangles.

48. What is the distance across corners of a hexagonal nut 1 in. on a side? What is the distance across the flats of the same nut?

49. Find the cost at 18 cts. per ft. of enclosing 1800 sq. ft. of land in the form of a regular hexagon.

50. Change 9 miles to rods; to yards; to feet; and to inches.

51. How many pieces can be cut from a brass rod 5 ft. long if the pieces are each (*a*) 2 in. long; (*b*) $2\frac{1}{2}$ in. long; (*c*) $5\frac{3}{4}$ in. long; (*d*) $6\frac{1}{8}$ in. long?

52. Five pieces of steel bar stock have the following lengths: 5 ft. 3 in., 7 ft. $2\frac{1}{2}$ in., 3 ft. 6 in., 9 ft. $3\frac{1}{4}$ in., 8 ft. 7 in. Find the total length of stock.

53. With $3\frac{1}{2}$-in. pipe weighing 9 lb. per ft., what is the total weight of pipe needed for a job requiring

5 pieces each	6 ft. 3 in.	long
3 pieces each	4 ft. 6 in.	long
2 pieces each	3 ft.	long
1 piece	5 ft. 8 in.	long
4 pieces each	10 ft. 4 in.	long

54. If a steel rail 29 ft. long weighs $676\frac{2}{3}$ lb., what is its weight per yd.?

55. A machine shop has a floor area of 23,870 sq. ft. If the width is 70 ft. what is its length? If it is desired to increase the floor area 10,000 sq. ft. by an extension 70 ft. wide, what will be the total length of the machine shop with the addition? Express answer in ft. and in.

56. What is the area in sq. in. and sq. ft. of a piece of boiler plate 109 in. long and 62 in. wide?

57. How many pieces of tin 4 in. × 6 in. can be cut from a tin plate 4 ft. × 1½ ft., making no allowance for material wasted in cutting?

58. If a gallon of paint covers 300 sq. ft., what will be the cost at $1.80 per gal. to paint a surface of 96 sq. yd.?

59. At 40 cts. per sq. yd., what will be the cost of plastering the sides and ceiling of a room 9 ft. wide, 12 ft. long and 8 ft. high?

60. What is the weight of a steel bar 3 ft. long, 4 in. wide and 2 in. thick? How much heavier is it than the same volume of water?

61. If steel rail weighs 70 lb. per yd. of length, how many tons of rail will it take to lay 1 mile of double track?

62. How many pieces of bar stock each 9 ft. long are required to make 180 spindles each 6½ in. long, if ½ in. is wasted for each spindle in cutting and finishing?

63. How many cubic inches in an iron casting weighing 746 lb.? How much heavier would be the same volume of lead?

CHAPTER IX

THE CIRCLE

37. Definition.—A circle is a plane figure enclosed by a curved line every point of which is equally distant from a point within called the center.

38. To Find the Circumference.—The circumference or distance around a circle is equal to 3.1416 times the diameter or distance through the center. Hence to find the circumference of a circle when we know the

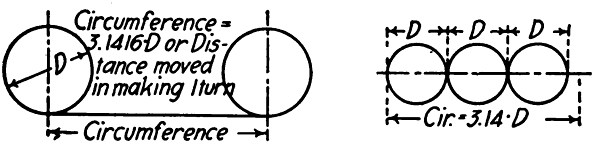


FIG. 67.

diameter *multiply the diameter by* 3.1416, or if C = the circumference and D = the diameter, then $C = 3.1416 \times D$.

If we know the circumference and desire to find the diameter we *divide* the *circumference* by 3.1416 and the result will be the diameter.

Using the same notation as above the rule may be written:

$$D = \frac{C}{3.1416}$$

The constant 3.1416 is indicated by the symbol π (Pi).

Using this abbreviation, the rule for the circumference of a circle may be written: Circumference = $\pi \times$ the diameter. The *radius* of a circle is one-half the diameter. In calculating the speed of cars and locomotives we use the circumference of the wheels or drivers, since, for example, a locomotive with an 80-in. driver advances for each turn of the drivers a distance equal to the driver circumference or $\pi \times 80$ or $3.14 \times 80 = 251.2$ in., or

$$\frac{251.2}{12} = 20.9 \text{ ft.}$$

PROBLEMS

1. Find the circumference of each of the following sizes of driving wheels: 48 in., 56 in., 60 in., and 80 in.

2. Which would travel the farther in making one turn, a 60-in. driver or an 80-in. driver? How much farther does one go than the other? (The distance each wheel goes in one turn is equal to its circumference.)

3. The 72-in. drivers on a locomotive make 265 turns per min. How many feet will the locomotive go per min.? (Multiply the circumference by the number of turns per min.)

How many feet will it go per hr.? (Multiply the number of feet gone over per min. by 60.)

How many miles will it travel per hr.? (Divide the number of feet traveled per hr. by 5280.)

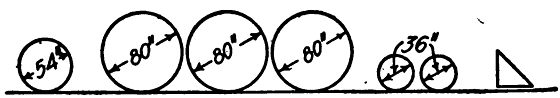


Fig. 68.

4. How many turns will each of the wheels on a locomotive make in traveling 1 mile if the front truck wheels are 36 in. in diam., the drivers are 80 in., and the trailing wheels are 54 in. in diam.? (Divide the number of feet in 1 mile (5280) by the circumference of each of the wheels in ft.)

5. Find the diameter of a driving wheel measuring 15 ft. 8½ in. around the outside.

6. The barrel of a boiler is 90 in. in diam. What length of sheet is required to make this barrel?

7. Good practice says that any point in the rim of a fly-wheel should not travel over a mile a minute. What then should be the maximum speed, in turns per min., of a fly-wheel 8 ft. in diam.?

8. The circular track, Fig. 69, carrying a turntable is 75 ft. in diam. How many feet of rails are required for this track.

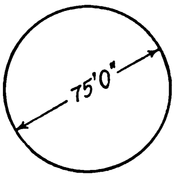


FIG. 69.

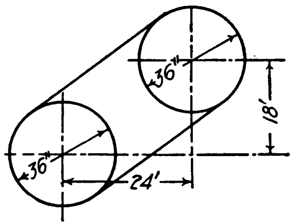


FIG. 70.

9. A line shaft is 18 ft. above and 24 ft. to the right of an engine shaft. If the two shafts are connected by a belt running over two 36-in. pulleys, as shown in Fig. 70, find the length (nearest foot) of belting required.

10. Figure 71 shows a pulley 3½ ft. in diam. which makes 200 turns per min.; through how many feet of space does the rim of

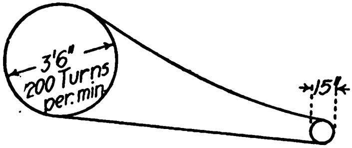


FIG. 71.

this pulley travel in 1 min.? If a belt passes from this pulley over a 15-in. pulley, through how many feet will the rim of the 15-in. pulley pass in 1 min.? How many turns per min. will the

15-in. pulley make? (Divide the belt travel by the circumference of the small pulley and the result will be the number of turns the small pulley makes per min.)

11. (*a*) How many turns a minute must a 1-in. diam. drill, Fig. 72, make so that its cutting speed at the outer edge of the lip will be about 30 ft. per min.? (*b*) With the drill running at the speed found and a feed of .01 in. per revolution, how long would it take to drill through a steel bar 4 in. thick?

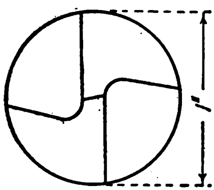


FIG. 72.

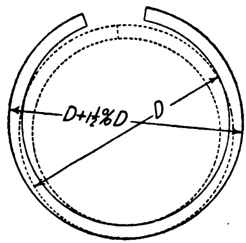


FIG. 73.

12. An axle 8 in. in diam. is being turned in a lathe. How many turns a minute must it be run so that the cutting speed will not exceed 60 ft. per min.?

13. Piston rings are turned to a diameter 1½ per cent. larger than the diameter of the cylinder barrel and then cut so that they can be sprung into place. To what diameter must the rings be turned for each of the following sizes of cylinders and how long a piece must be cut from each so that after the ring is sprung in place it will have $\frac{1}{16}$ in. clearance between ends.

Diam. of cylinder	Diam. of ring	Length of piece cut out
20	$20\frac{5}{16}$	$1\frac{1}{32}$
21		
22		
24		
26		

Solution:

20 in. = diam. of piston.

$1\frac{1}{2}\%$ = .015 × 20 in. = .30 in. increase.

.3 in. $= .3 \times 32 = \frac{9.6}{32} = \frac{10}{32}$ in. or $\frac{5}{16}$ in. increase.

20 + $\frac{5}{16}$ in. = $20\frac{5}{16}$ = diam. of ring.

$\frac{10}{32} \times 3.1416 = \frac{31.416}{32} = \frac{31}{32}$ increase in circumference.

$\frac{1}{16}$ in. = $\frac{2}{32}$ clearance.

$\frac{31}{32} + \frac{2}{32} = \frac{33}{32}$ or $1\frac{1}{32}$ in. to be cut out.

39. Areas of Circles.—If we take a circular board and divide it into sections as shown in Fig. 74, and place them as in Fig. 75, we practically have a rectangle

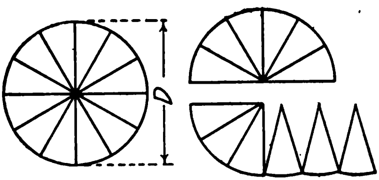


Fig. 74.

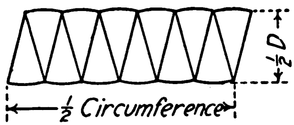


Fig. 75.

whose length is ½ the circumference and whose width is ½ the diameter of the circle.

Hence to find the area of the circle we multiply ½ the diameter by ½ the circumference. If we let D = the diameter and C = the circumference, then the area equals $\frac{1}{2}D \times \frac{1}{2}C$ or $D \times \frac{1}{4}C$. Since the circumference of a circle equals the diameter × 3.1416 we may write the above rule as follows:

$$\text{Area} = D \times \tfrac{1}{4}D \times 3.1416$$
$$= D^2 \times \frac{3.1416}{4} = .7854D^2$$

Hence to find the area of a circle: Square the diameter and *multiply by ¼ of* 3.1416 *or* .7854. Using the symbol π for 3.1416 we have

$$\text{Area of a circle} = \frac{\pi D^2}{4} = \frac{3.1416D^2}{4} = .7854D^2$$

PROBLEMS

14. Find the area of each of the following size pistons: 20 in., 21 in., 23 in. in diam.

15. An air drum is 18 in. in diam. What is the area of each end in sq. in.? If the pressure per sq. in. is 75 lb., find the total pressure tending to blow out the end. (Multiply the number of square inches in one end by the pressure per sq. in.)

16. The piston of a locomotive is 24 in. in diam. Find its area in sq. in. If the highest pressure carried is 205 lb. per sq. in., what would be the total pressure tending to blow off the cylinder head?

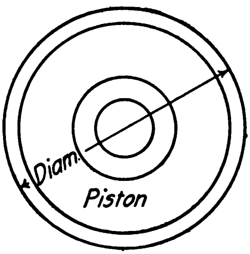


FIG. 76.

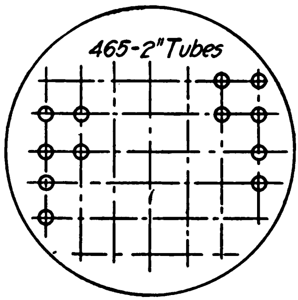


FIG. 77.

17. The diameter of the front head of the boiler in Fig. 77 is 76¾ in There are 465 tubes whose outside diameter is 2 in. What is the area in sq. in. of the front tube sheet after deducting the area taken up by the ends of the tubes?

18. If the inside diameter of the tubes in Prob. 17 is 1.81 in., what is the total fire area through these tubes?

19. What is the total fire area through the tubes of a boiler if there are 173 1¾-in. tubes whose inside diameter is 1.56 in.?

20. The diameter of the piston on a stationary engine is 22 in. If the piston rod diameter is 3 in., what is the net area of the piston on the back side?

21. Which has the greater capacity, one 8-in. pipe or four 2-in. pipes, supposing their capacity to be proportional to their respective areas?

22. How many 6-in. pipes would be required to replace one 12-in. pipe so that the capacity would be the same?

23. Find the area of a 4-in. circle, of an 8-in. circle, of a 12-in. circle.

The area of an 8-in. circle is how many times the area of a 4-in. circle? The area of a 12-in. circle is how many times the area of a 4-in. circle?

24. Find the area at the root of the thread for the following sizes of bolts:

Diam. of bolt	Diam. at root	Area at root of thread
1 in.	.837 in.	
1⅛ in.	.940 in.	
1¼ in.	1.065 in.	
1½ in.	1.284 in.	

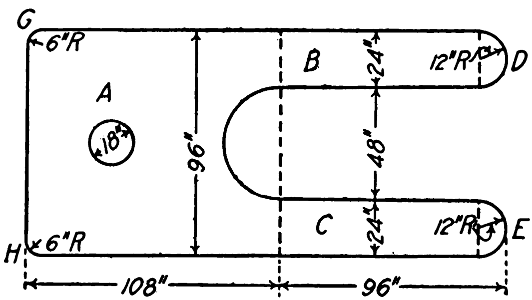


Fig. 78.

25. Figure 78 shows the top sheet of a tank made of ¼-in. steel plate. Find its area by dividing it into rectangles as shown by the dotted lines.

Find the area of the rectangles A, B, and C as indicated by the dotted lines. Subtract the area of an 18-in. circle and ½ the area of a 48-in. circle. Add the area of a 24-in. circle to allow for rounded heads at D and E. To correct for corners G and H subtract twice the difference between the area of a 6-in. square, Fig. 79, and ¼ the area of a 12-in. circle (6-in. radius). Find the weight of the sheet if ¼-in. steel weighs 10 lb. per sq. ft. of surface.

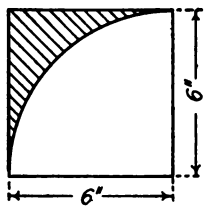


FIG. 79.

40. Finding the Diameter when the Area is Known.—Sometimes we know the area of a circle and wish to find the diameter. This operation is the reverse of that of finding the area when we know the diameter, the same as finding the square root is the reverse of "squaring." To find the diameter from the area, *divide the area by* .7854 *and extract the square root of this quotient.* If A = the area and D = the diameter, then

$$D = \sqrt{\frac{A}{.7854}}$$

Example.—Find the diameter of a circle whose area is 12.57 sq. in.

Rule.—Diameter $= \sqrt{\frac{\text{area}}{.7854}}$

$$= \sqrt{\frac{12.57}{.7854}} = \sqrt{16.00} = 4 \text{ in.}$$

PROBLEMS

26. Find the diameter of a circle whose area is 19.635 sq. in.

27. The area of a piston is 254.5 sq. in. What is its diameter?

28. The total pressure in a brake cylinder is to be 5670 lb. If the pressure per sq. in. is 50 lb., what must be the area of the piston? (Divide the total pressure by the pressure per sq. in. to get the area.) What is the diameter of this piston?

29. A 12-in. pipe is to be branched off into two equal pipes. What must be the diameter of these pipes so that their combined area is the same as that of the 12-in. pipe?

30. A rectangular fan outlet 48 in. × 36 in., Fig. 80, is to be connected to a circular duct so that the area through the duct is the same as the area of the fan outlet. What must be the diameter of the circular duct?

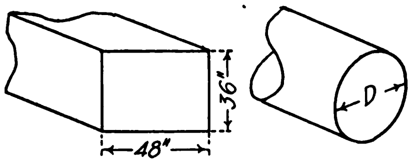


FIG. 80.

31. A circular duct in a heating system is to supply air for five rectangular outlets 8 in. × 10 in. What must be the diameter of the duct so that its capacity will be equal to the combined capacity of the five outlets?

32. Four stationary steam engines are supplied with steam from one steam "header." Each engine is supplied with steam through a 6-in. pipe. What must be the diameter of the steam header so that its capacity will be equal to that of the combined capacity of the four 6-in. pipes? (Make the area of the header equal to the combined area of the four 6-in. pipes.)

33. What is the diameter of each of the pistons having the following areas: 63.6 sq. in., 78.54 sq. in., 113.1 sq. in., and 84 sq. in.?

34. The total pressure on the piston in a brake cylinder is to be 6780 lb. What must be the diameter of the cylinder if the pressure per sq. in. is to be 50 lb.?

35. In order to give the joint in Fig. 81 the proper strength it is found that the rivets must have a cross-sectional area of .6 sq. in. What must be their diameter?

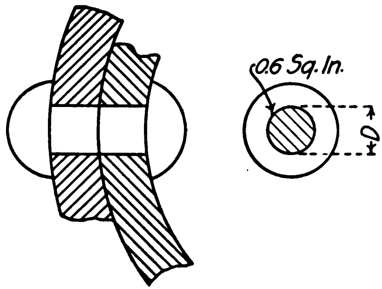


Fig. 81.

36. The fire area through the tubes of a locomotive is to be 6.25 sq. ft. If there are to be 373 tubes, what must be the inside diameter of each tube? If the thickness of the metal in these tubes is $\frac{1}{8}$ in., what standard size tube would you use?

37. The high-pressure cylinder of a Corliss engine is 18 in. in diam. The low-pressure cylinder has twice the area of the high-pressure cylinder. Find the diameter of the low-pressure cylinder.

38. How far does a 72-in. driver travel in making 100 revolutions?

39. Find the circumference, area in sq. in., volume in cu. in., and weight in lb. of a $\frac{3}{8}$-in. steel plate 24 in. in diam.

40. How much more will it cost to enclose 1600 sq. ft. of land in the form of a square than in the form of a circle with fencing at 18 cts. per ft.?

41. From a piece of sheet iron 3 ft. long and 2 ft. wide are cut four discs each 10 in. in diam. How many square inches of the sheet are left if no allowance is made for loss in cutting the discs?

42. It is desired to purchase a site for a roundhouse to accommodate twenty locomotives. For each locomotive an area of 1000 sq. ft. is allowed. What must be the diameter of the circular plot required?

43. If a locomotive has 82-in. drivers and is making 200 r.p.m., what is its speed in miles per hr., allowing for a 2% slip?

44. Good practice calls for a "piston speed" of 600 ft. per min. for stationary Corliss engines. At what speed in revolutions per min. would the following stroke engines be rated: 18-in. stroke, 24-in. stroke, 36-in. stroke, 42-in. stroke?

45. A disc saw that is 14 in. in diam. is to have forty-four teeth. What distance apart are they on the circumference and give the angular distance between the teeth?

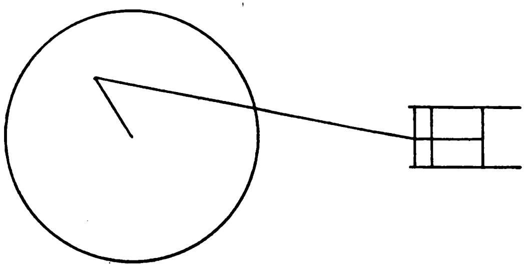


Fig. 82.

46. Find the circumference, area in sq. in., volume in cu. in., and weight in lb. of a ¼-in. steel plate 3 ft. in diam.

47. In Fig. 82 the distance from the center of the crank pin to the center of the driving wheel is 18 in. What is the piston travel or stroke of the engine? If the driving wheel turns 150 times per min., find the corresponding piston speed in ft. per min.

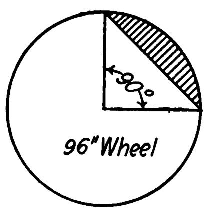


Fig. 83.

48. What length of iron strip is required to make eight bands for a tank 6 ft. 4 in. in diam., if each band laps 4 in.?

49. Calculate the number of cubic inches in the counter-balance shown in Fig. 83, if the metal is 5 in. thick. What is its weight at an average of .30 lb. per cu. in.?

50. A casting has a cored hole 4 in. in diam. and 20 in. long. How much will this hole reduce the weight of the casting if the metal weighs .26 lb. per cu. in.?

51. A 14-in. water pipe is branched into three equal pipes so that the combined capacity of the three pipes is equal to that of the 14-in. pipe. If the amount of water carried is considered as dependent on the area of cross section of the pipes, how large must be each of the three pipes?

52. A weight is held by three rods each 1 in. in diam. If the same weight is to be held by one rod what must be its diameter to be as strong as the three rods together?

53. A hollow cast-iron column has inside and outside diameters of 12 in. and 16 in. respectively. If the column is 12 ft. long, what is its weight?

54. Find the net area (cross-sectional portion) of the steel plate shown in Fig. 84. Find the number of cubic inches in the plate if it is 3/8 in. thick. What is its weight at .28 lb. per cu. in.?

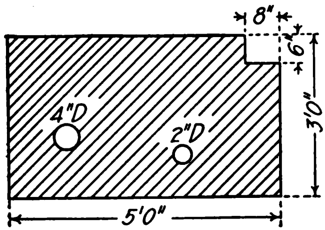


FIG. 84.

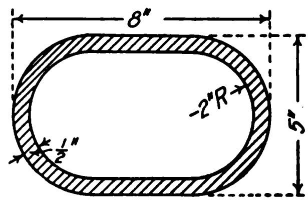


FIG. 85.

55. Find the area in sq. in. of the cross-sectional portion of the steam pipe shown in Fig. 85.

55a. Find the area of the eccentric cross section shown in Fig. 86.

55b. Find the area of the ring section shown in Fig. 87.

56. Find the circumference of locomotive drivers 72 in., 78 in., 80 in., and 82 in. in diam.

57. What distance would a locomotive travel in 1 min. for each of the above cases if the number of revolutions of the driver were 200 per min.?

58. If a locomotive has 82-in. drivers, how many revolutions will each driver make in 1 mile if there is no slipping?

59. When a locomotive went from Altoona, Pa., to Harrisburg, Pa., a distance of 131.1 miles, in 112 min., how many revolutions per min. did the drivers make if there was no slipping? How many revolutions did they make per sec.? Diameter of driver is 80 in.

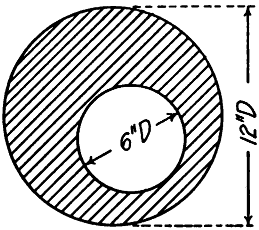


FIG. 86.

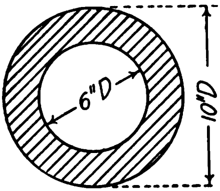


FIG. 87.

60. A pulley is 3 ft. in diam. and makes 220 r.p.m. How fast is the rim of the pulley traveling?

61. The drivers on a locomotive are 80 in. in diam. How many miles per hr. will the locomotive travel when the drivers make 275 turns a min.?

62. If the diameter of a circle is five times that of another, how do their areas compare?

63. Which will carry the more water, other things being equal, four 1-in. pipes or one 4-in. pipe? Explain your answer fully and show all the calculations necessary.

64. The total pressure acting on a piston is 6284 lb. with a pressure per sq. in. of 200 lb. What is the diameter of the piston on which the pressure acts?

65. If a shaft 6 in. in diam. is being turned on a lathe, at how many turns a minute must it be run so that the cutting speed will not exceed 60 ft. a min.?

66. What length of belt is required to connect two 24-in. pulleys whose centers are 8 ft. apart?

67. What is the total pressure in lb. acting on a 10-in. diam. piston with a pressure of 80 lb. per sq. in.?

68. A pulley 2 ft. in diam. makes 200 r.p.m. At what speed is the rim of the pulley traveling?

69. How many revolutions per min. must an 8-ft. diam. fly-wheel make so that the surface speed of the rim will not exceed a mile a min.?

70. How many 2-in. pipes are required to carry the same amount of water as a 6-in. pipe if we consider the amount of water carried to depend entirely on the area of cross section of the pipes?

71. How large a piston is required to obtain a total pressure of 45,000 lb. with a pressure of 200 lb. per sq. in.?

CHAPTER X

MENSURATION OF SIMPLE SOLIDS

41. Volume of Solids of Rectangular Section.—In our work on mensuration of surfaces we used two measurements, length and width. Surfaces were measured in square feet and square inches. If we wish to measure the volume or contents of solids or liquids we must take into account not only the length and width but also the thickness.

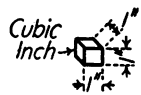


FIG. 88.

Figure 88 shows a block or cube 1 in. on each side. This is called a cubic inch. Fig. 89 shows a cubic block 1 ft. each way, known as a cubic foot.

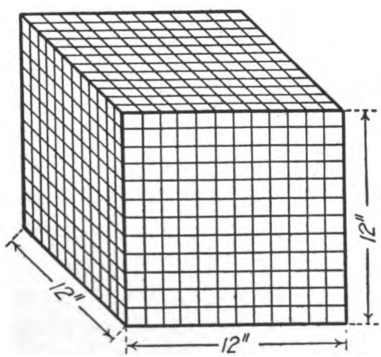


FIG. 89.

Contents or volumes of solids are measured in cubic inches, cubic feet, cubic yards, etc.

On examining Fig. 90 it is seen that it contains 144 cu. in. since it is 12 in. long, 12 in. wide, and 1 in. thick. It has the same volume or contents as each of the layers or sections shown in Fig. 89 which consists of 12 such layers or sections. The cubical solid shown in Fig. 89, therefore, contains 12 × 144 or 1728 cubes 1 in. on a side or 1728 cubic inches (cu. in.).

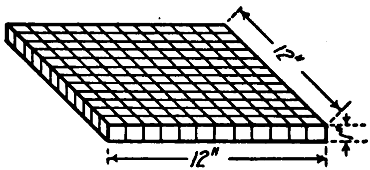


Fig. 90.

The following should be kept clearly in mind:
One foot = 12 in. (the measure of length).
One square foot = 144 sq. in. (the measure of area).
One cubic foot = 1728 cu. in. (the measure of volume).

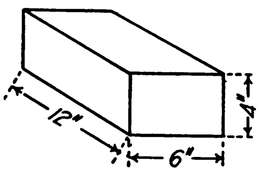


Fig. 91.

The contents of any rectangular solid or block whose length, width, and thickness are known may be found by multiplying together the length, the width, and the thickness. Thus the volume of a block of steel 6 in. wide, 4 in. thick, and 12 in. long, Fig. 91, is 6 in. × 4 in. × 12 in. or 288 cu. in. Since the width times the thickness gives the area of the end, it may be well to note that *the volume* of any solid whose cross section is the same for its entire length *equals the area of one end times the length,* or volume = area of end × length. This rule will hold no matter what may be the shape of the cross section.

42. Volume of Cylinders.—The volume of a cylinder, Fig. 92, 24 in. in diam. and 26 in. long is found by multiplying the area of a 24-in. circle by 26 in.; thus 24 × 24 × .7854 = 452.39 sq. in., area of end. 452.39 × 26 = 11,762.14 cu. in. volume.

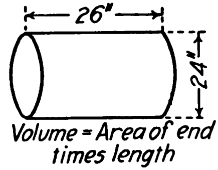


Fig. 92.

The following problems are worked according to the foregoing rules:

PROBLEMS

1. How many cubic inches of metal in a wrought-iron bar 2 in. wide, 1½ in. thick, and 52 in. long?

2. A tank is 8 ft. long, 6 ft. wide, and 5 ft. deep. How many cubic feet does it contain?

3. How many cubic feet in a steel tank car 8 ft. in diam. and 33 ft. long?

4. Find the number of cubic inches in a rough forging for an axle 9 in. in diam. and 5 ft. 11 in. long.

5. A tank car is 33½ ft. long and 8½ ft. in diam. How many cubic feet does it contain? How many cubic inches? How many gallons of oil would it hold, each gallon containing 231 cu. in.?

6. An engine has a cylinder 24½ in. in diam. with a 26-in. stroke. Find the piston displacement. (Find the volume of a cylinder 24½ in. in diam. and 26 in. long.)

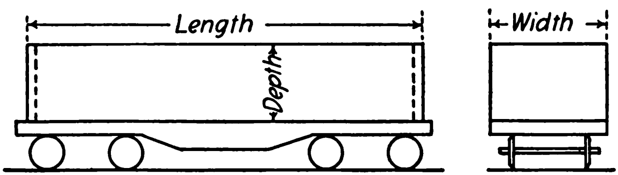


Fig. 93.

7. Find the capacity in cu. ft. of gondola freight cars having inside dimensions as given in the following table and according to Fig. 93:

Length	Width	Depth	Capacity
37 ft. 6 in.	8 ft. 8 in.	2 ft. 6 in.	
38 ft. 3 in.	9 ft. 3 in.	3 ft. 9 in.	
37 ft. 8 in.	9 ft. 3 in.	3 ft. 9 in.	
40 ft. 6 in.	8 ft. 9 in.	2 ft. 6 in.	

Weights of Solids.—The weight of any solid, as a block of iron, may be found from its volume when we know its weight per cu. in. or per cu. ft. For example, since the weight of steel is .28 lb. per cu. in. the weight of a rough steel forging for a crank pin $5\frac{1}{2}$ in. in diam. and 12 in. long may be found as follows:

Area of end $= 5.5 \times 5.5 \times .7854 = 23.76$ sq. in.
Volume $= 23.76 \times 12 = 285.12$ cu. in.
Weight $= 285.12 \times .28 = 79.8+$ lb.

43. Table of Weights.—The following table giving weights of different materials commonly used is to be employed in working out the following problems:

Material	Wt. per cu. in., lb.	Wt. per cu. ft., lb.
Cast iron	.26	450
Wrought iron	.278	480
Steel, cast	.28	481
Steel, rolled	.2833	490
Copper	.322	556
Bronze	.319	550
Brass	.3	523
Coal (bit.)	....	53 approx.
Coal (anthr.)	....	60 approx.
Sand (dry)	....	100 approx.
Sand (wet)	....	120 approx.
Concrete	....	125 approx.

PROBLEMS

8. Find the weight per ft. of length for the following sizes of bar wrought iron: 1 in. square, 1½ in. square, and 2 in. × 1 in.

9. Find the weight per ft. of length for the following sizes of round wrought-iron bars: 1 in. diam., 1½ in. diam., and 2 in. diam.

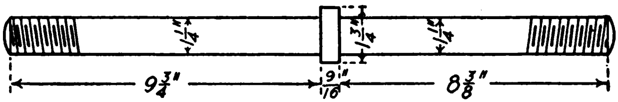


FIG. 94.

10. Find the weight of cylinder and saddle shoulder bolt in Fig. 94.

11. Find the weight of rough forgings for crank pins, as per Fig. 95 and table. Their diameter is the same throughout their entire length. Allow ¼ in. for finish all over.

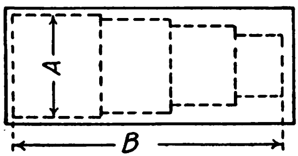


FIG. 95.

A	*B*	Weight
8¼ in.	23½ in.	
5¼ in.	15¼ in.	
4 in.	12¾ in.	

12. Find the weight of the finished crank pin in Fig. 96.

13. Find the weight of the fire-box side sheet in Fig. 97.

14. Find the weight of the side sheet in Fig. 98.

15. Find the weight of a steel block 1 ft. long, 2 in. wide and 1½ in. thick.

16. Find the relative weights of two pieces of timber, one white pine 4 in. by 3 in. and 12 ft. long and weighing 28 lb. per cu.

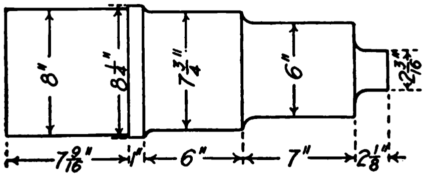


FIG. 96.

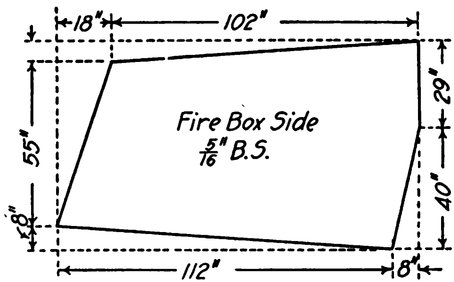


FIG. 97.

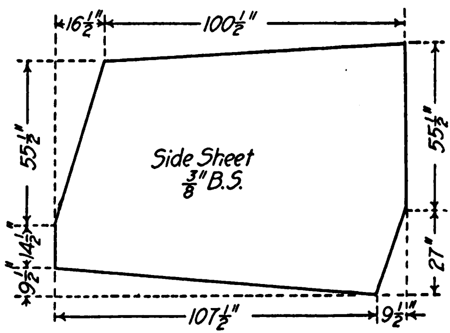


FIG. 98.

ft., the other red oak 4 in. by 4 in. and 10 ft. long, weighing 46 lb. per cu. ft.

17. How many tons of coal can be stored in a cubical coal bin 21 ft. on a side? (One ton occupies about 37 cu. ft.)

18. Find the weight of a 10-ft. steel rod 1 in. in diam. Find the weight of a steel rod the same length but twice the diameter. How does doubling the diameter affect the weight?

19. Find the capacity in gal. of the tank car shown in Fig. 99. If the car is loaded with oil .94 as heavy as water, what weight of oil does the tank carry?

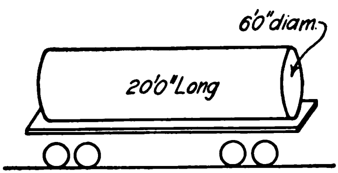


Fig. 99.

20. The volume of a cylindrical tank whose height is 8 ft. and diam. 3 ft. is what proportion of the volume of a second tank of the same height with base 4 ft. in diam.?

21. Find the weight of a 6-ft. iron pipe, inside diam. 3½ in., outside diam. 3⅞ in., if the iron weighs 480 lb. per cu. ft.?

22. Find the number of cubic feet in a tank whose inside diam. is 8 ft. and whose height is 12 ft. How many gallons of water does this tank hold?

23. How many cubic inches in a 1-in. steel rod 5 yd. in length? What is the weight of the rod?

24. A steel shaft weighing 850 lb. is 22 ft. long. What is its diameter?

25. Find the number of cubic inches in an air reservoir 28 in. in diam. and 48½ in. long.

26. How many cubic feet of water are pumped per min. in a 24 in. × 30 in. pump making 20 strokes per min., with no allowance for leakage? (24 in. × 30 in. means 24-in. diam. and 30-in. stroke.)

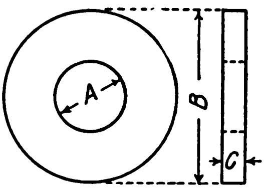
27. A water tank is cylindrical in shape and has a diameter of 10 ft. and a height of 14 ft. How many gallons does it hold when full?

28. What is the weight of a steel shaft 3 in. in diam. and 20 ft. long, if the steel weighs 488 lb. per cu. ft.?

29. Find the difference in weight of a 2-in. steel rod 10 ft. long and a 1-in. cast-iron rod 8 ft. long.

30. How high must be a cylindrically shaped water tank 12 ft. in diam. to hold 12,000 gal.?

30a. The following table shows the dimensions for four different sizes of washers. In this table "*D*" is the diameter of the bolt on which the washer works. The other dimensions have the significance as shown in Fig. 100.



D	*A*	*B*	*C*
in.	in.	in.	in.
1/4	5/16	5/8	1/16
1/2	9/16	1 1/4	3/32
3/4	13/16	1 3/4	1/8
1	1 1/16	2 1/4	5/32

Fig. 100.

For each of the above sizes find the weight of 1000 washers, the material being wrought iron weighing .278 lb. per cu. in.

31. If a water tank 6 ft. inside diam. and 8 ft. high is full of water, what pressure is produced on the supporting structure due to the weight of water alone?

32. Find the weight of a 20-ft. hollow, cylindrical, cast-iron column if the outside and inside diameters are 20 and 14 in. respectively.

33. A steel shaft weighing 844.46 lb. has a length of 20 ft. What is its diameter?

34. Twenty 3/8-in. holes are drilled in a 1/2-in. steel plate. How much will the holes reduce the weight of the plate?

35. Find the height of a cylindrically shaped tank 10 ft. in diam. to hold 7480 gal.

36. What is the length of a steel shaft 2 in. in diam. and weighing 108 lb.?

37. What is the volume in cu. in. and also in cu. ft. of a steam cylinder 24 in. $\times$ 30 in.?

44. Board and Timber Measure.—Timber is bought and sold by the thousand board feet, abbreviated

(M.B.F.). To calculate the number of feet board measure in a board or piece of square timber, multiply together the length in feet, the width in feet and the thickness in inches. For example, a piece of timber 16 ft. long, 8 in. wide and 4 in. thick contains $16 \times 8/12 \times 4$ or 42.67 board ft. In other words, *one board foot* is 1 ft. square and 1 in. thick as shown in Fig. 102, and therefore contains $1/12$ cu. ft. volume.

In a cubical piece of timber 1 ft. on a side there are therefore 12 board ft., as shown in Fig. 101; in other

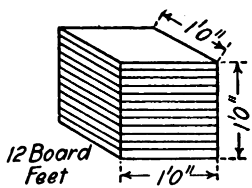


FIG. 101.

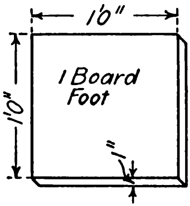


FIG. 102.

words, the number of board feet equals the number of cubic feet multiplied by 12.

Lumber usually comes in lengths of an even number of feet, such as 12, 14, 16, and 18, and when less than 1 in. in thickness is counted as if 1 in. thick in figuring up the bill.

In estimating the number of shingles required for a given job the calculation is made by the "square." One "square" being an area 10 ft. by 10 ft. or 100 sq. ft. Shingles are cut 16 in. or 18 in. in length and are laid from 4 in. to 6 in. exposed to the weather. They are sold in bundles of 250 or 4 bundles per thousand. A shingle is counted as 4 in. wide and as stated from 4 in. to 6 in. exposed to the weather. On this basis the following figures may be used for calculating shingles required.

Inches exposed to weather	Area covered by 1000 shingles
4	100 sq. ft.
5	135 sq. ft.
6	158 sq. ft.

PROBLEMS

38. Find the number of board feet in a timber 18 ft. long, 10 in. wide, and 3 in. thick.

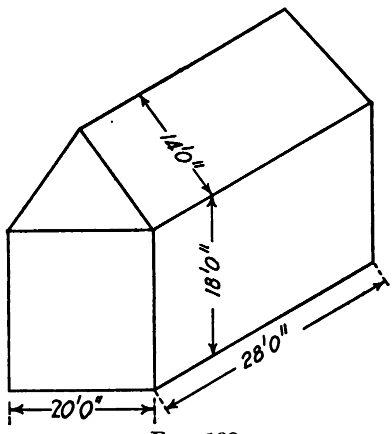


Fig. 103.

39. How many board feet in a timber 16 ft. long, 2 in. thick and widths at its ends of 10 in. and 14 in.?

40. Find the amount of lumber required to lay a floor in a room 20 ft. by 14 ft. allowing 10% for waste?

41. It is desired to build a shop fence 8 ft. high and 500 ft. long. The boards are to be of 1-in. material and are nailed to two 2-in. × 4-in. rails. How many thousand board feet (M.B.F.) are required allowing 10% for waste?

42. Find the cost of shingling a roof 40 ft. × 30 ft. with shingles laid 5 in. to weather and costing $4.50 per thousand.

43. Find the cost of flooring a machine shop 80 ft. by 100 ft. with 2-in. planks costing $32 per thousand.

44. Find the cost of the following bill of lumber:

10 joists, 2 in. × 8 in. and 16 ft. long at $28 per thousand.
4 joists, 3 in. × 8 in. and 12 ft. long at $30 per thousand.
500 feet hemlock sheathing at $26 per thousand.
10 bundles laths at 40 cts. per bundle.
4 floor beams, 2 in. × 7 in. and 18 ft. long at $34 per thousand.

45. Find the cost of sheathing the building shown in Fig. 103 at $28 per M.B.F., allowing 200 board ft. for windows and adding 12% for waste. The calculation is to include the four sides, gables and roof.

CHAPTER XI

USE OF RULES AND FORMULAS
GENERAL EXPRESSIONS

45. Use of Letters to represent Quantities.—It often becomes convenient in mathematical work to represent numbers or the amount of an unknown number of objects by letters of the alphabet in addition to the ordinary figures. The letters of the alphabet are used as general symbols of numbers to which any particular values may be given.

Thus, we know the circumference of a circle is equal to 3.1416 multiplied by its diameter. Hence we may write 3.1416 multiplied by D or $C = 3.1416 \times D$. Here C represents the circumference and D the diameter of the circle and both are general terms applying to *all sizes* of circles. In any problem, however, a letter is understood to have the same value throughout the problem. The quantity which a letter represents is called the value of the letter, or its *numerical value.*

The signs used in connection with the letters to denote addition, subtraction, multiplication, division, squaring and square root are the same as those which are used in arithmetic.

46. The Sign of Addition [+].—The sign + is read "plus." Thus, $3 + 2$ is read "3 plus 2" and indicates that the number 2 is to be added to 3; $a + b$ is read "a plus b" and indicates that the number b is to be added to a.

Indicate the following sums:

1. a plus b; c plus d;
2. a plus b increased by c; c increased by d plus a;

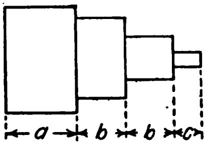


FIG. 104.

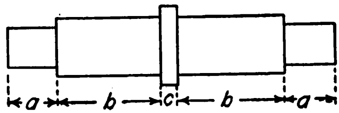


FIG. 105.

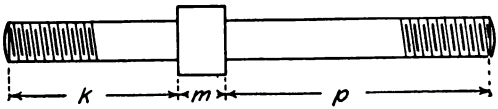


FIG. 106.

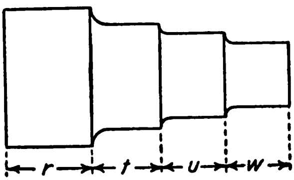


FIG. 107.

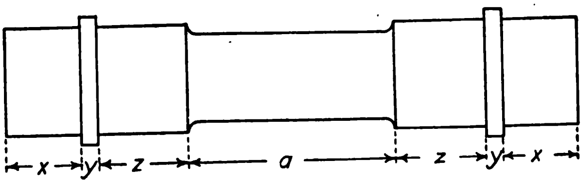


FIG. 108.

3. x plus y; x plus y plus z; a increased by b plus c;
4. x plus y increased by z.
5. What is the length of the casting in Fig. 104? (Indicate the sum of the different lengths.)
6. What is the length of the casting in Fig. 105?

7. Find the length of the cylinder and saddle shoulder bolt in Fig. 106.

8. Find the length of crank pin in Fig. 107.

9. Find the length of axle in Fig. 108.

47. The Sign of Subtraction [−].—The sign − is read "minus." Thus 3 − 2 is read "3 minus 2" and indicates that the number 2 is to be subtracted from the number 3; $a - b$ is read "a minus b," and indicates that the number b is to be subtracted from the number a.

10. Indicate the following: a minus b; b diminished by c; a minus b minus c; a minus b diminished by c; x diminished by y minus z.

11. If l is the length of the cylinder and saddle shoulder bolt in Fig. 109, what is the length of the shoulder?

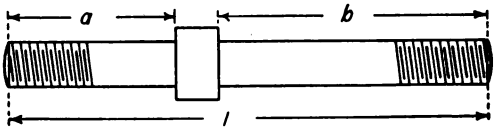


FIG. 109.

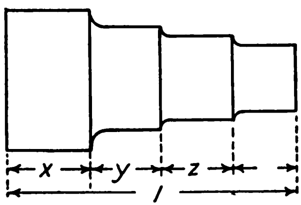


FIG. 110.

12. If l is the length of crank pin in Fig. 110, what is the length of the last step?

13. If a is the total length of the axle in Fig. 111, how much of the length is taken up by the wheels?

48. The Sign of Multiplication [×].—The sign × is read "times" or "multiplied by."

Thus 3×2 is read "3 times 2," and indicates that the number 3 is to be multiplied by 2; $a \times b$ is read "a times b," and indicates that the number a is to be multiplied by the number b.

14. Indicate the following products: a times c; b times x; x multiplied by y; x times y multiplied by z; a multiplied by b times y.

15. If a man works x hr. per day and handles y castings per hr., how many castings does he handle each day?

16. If there are an average of "a" trains leaving Altoona, Pa. every day and an average of c cars per train, how many cars leave Altoona per day?

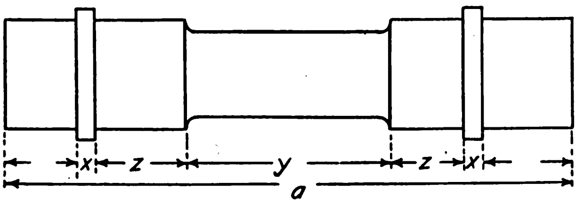


FIG. 111.

49. The Sign of Division [÷].—The sign ÷ is read "divided by." Thus $6 \div 3$ is read "6 divided by 3," and indicates that the number 6 is to be divided by 3; $a \div b$ is read "a divided by b," and indicates that the number a is to be divided by the number b.

Division is also indicated by writing the dividend above the divisor with a horizontal line between them; or by separating the dividend from the divisor by an oblique line. Thus $\frac{a}{b}$ or a/b means the same as $a \div b$.

17. Indicate the following: a divided by b; c divided by d; ax divided by y; bc divided by a; abc divided by x.

18. If there are a cars in a railroad yard, how many trains will there be if there are to be an average of b cars per train?

52. Factors.—When a number is the product of two or more numbers, each of these numbers, or the product of two or more of them, is called a *factor* of the given number. Thus 2, a, b, $2a$, and ab are factors of $2ab$.

The sign $\times$ is omitted between factors if the factors are letters or a number and a letter. Thus, we write abc for $a \times b \times c$ and $23ab$ for $23 \times a \times b$.

If one or more factors of a product are equal to 0, the product is equal to 0. Thus, $abcd = 0$, if a, b, c, or $d = 0$, and $2abc = 0$, if a, b, or $c = 0$.

53. Coefficients.—Any factor of a product may be considered as the coefficient of the remaining factors.

Thus, in the expression $8a$, 8 is the coefficient of a; and in the expression ax, a is the coefficient of x.

If no numerical coefficient is written, 1 is understood. Thus, the expression ab is understood to have the coefficient 1, and is the same as if it were written $1ab$.

54. Roots.—A root is one of the equal factors of a number. If the number is broken up into *two equal factors*, each factor is called a *square root;* if it is broken up into *three equal factors*, each factor is called a *cube root;* if into *four equal factors*, each factor is called the *fourth root;* and so on. Thus, if we break 16 up into two equal factors, we have 4 and 4, and 4 is the *square root* of 16; if we break 8 up into three equal factors we have 2 and 2 and 2, and 2 is the cube root of 8, and so on.

The root sign is $\sqrt{\ }$. Except for the square root, a number symbol is written over the root sign to show into how many equal factors the given number is to be resolved or broken up. This number symbol is called the *index of the root.* Thus, $\sqrt{16}$ means the square root of 16; $\sqrt[3]{27}$ means the cube root of 27.

Indicate the following:

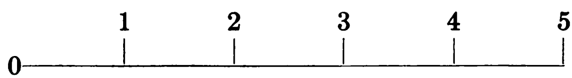
26. The square root of a.
27. The cube root of b.
28. The fifth root of x.
29. The third root of a plus b.
30. The fourth root of x minus y plus z.
31. The square root of x plus y multiplied by a.
32. The third root of a times b times the sum of a plus b divided by c.

55. Positive and Negative Numbers.—A number or expression with a positive sign + in front of it is called a positive number or expression. Thus, $+2$; $+4$; $+a$; $+b$; $+(a - b)2$, etc.

A number or expression with a negative sign − in front of it is called a negative number or expression. Thus, -3; -5; $-b$; $-c$; $-(a + b)2$, etc.

We are accustomed to think of these signs as meaning addition and subtraction, but they also have another meaning, that is to show the position of the number or expression relative to the zero point.

If from a given point, marked 0, we draw a straight line to the right, and beginning from the zero point lay off units of length on this line, the successive repetitions of the unit will be expressed by a series of numbers called the Natural Series of numbers, 1, 2, 3, 4, etc. Thus,

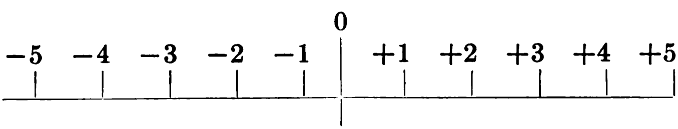


In this series, if we wish to add 2 to 5 we begin at 5, count 2 units to the right, and arrive at 7. If we wish to subtract 2 from 5, we begin at 5, count 2 units to the left, and arrive at 3. If we wish to subtract 5 from 5, we begin at 5, count 5 units to the left from 5, and arrive at 0. If, however, we wish to subtract 5 from 2, we cannot do it, because when we have counted to the left as far as 0, the series of numbers comes to an end. Then, in order to subtract a greater number from a smaller number it is necessary to assume a new series of numbers beginning at 0 and extending to the left. If the natural series advances from 0, to the right, by repetitions of the unit, the new series must recede from 0, to the left, by repetitions of the same unit, and the opposition between the right-hand series and the left-hand series must be clearly marked.

This opposition is indicated by calling every number to the right of the zero point a positive number and prefixing to it the sign +; and by calling every number to the left of the zero point a negative number and prefixing to it the sign −.

The positive sign + means that the value of the number or expression is above zero, or to the right in the series, and the negative sign − means that the number or expression is below zero, or to the left in the series.

The two series of numbers will be written thus:



and may be considered as forming but a single series consisting of a positive branch, a negative branch, and zero. If, now, we wish to subtract 5 from 2, we begin at 2 in the positive series, count 5 units in the negative direction (to the left) and arrive at −3 in the negative series, that is, $2 - 5 = -3$.

The following explanation may help the beginner to more easily understand the previous discussion. When keeping the score of a game, if a player is loser a number of points, he scores a minus or negative quantity. If his loss is more than he has won, or more than his total score, he has less than nothing, and his total score will be a negative quantity. Again, suppose a man contracts a debt of, say, \$50. He has no money to pay his debt and will have minus \$50 or \$50 less than nothing. Now, suppose that he receives \$30 and pays this amount on his debt. He adds \$30 to − \$50 and then has − \$20 or \$20 less than nothing, that is, he still owes \$20. Also in measuring temperature, degrees above zero may be called plus (+) and those below zero minus (−).

56. Double Meaning of the Signs + and −.—The use of the signs + and − to indicate addition and subtraction must not be confused with the use of the signs + and − to indicate in which series, the positive or the negative, a given number or expression belongs.

The Absolute Value of a number is its value independent of its sign. Numbers regarded without reference to the signs + or − are called absolute numbers. Thus, the absolute values of +4, +5, −2, −3, −6 are 4, 5, 2, 3, and 6.

The sign simply shows whether the number belongs to the positive or the negative series of numbers, and the absolute value shows the place the number occupies in the positive or the negative series. When no sign stands before the number, the sign + is always understood; but the sign − is never omitted.

57. Rules for Addition.—1. If the numbers have like signs, find the sum of their absolute values and prefix the common sign to the result. 2. If the numbers have unlike signs, find the difference of their absolute values and prefix the sign of the greater number to the result.

PROBLEMS

33.

$+21$	-42	-56	$+76$
$+36$	-65	$+22$	-98
$+57$	-107	-34	-22

Find the following sums:

34.

(1) $+19$	(2) $+36$	(3) $+41$	(4) $+32$	(5) $+29$
$+14$	$+10$	$+7$	-18	-15
(6) $+23$	(7) -56	(8) -66	(9) -101	(10) -436
-47	$+31$	-81	-372	-732

Addition of Monomials.—A *monomial* is an algebraic expression containing one term only. Thus, $7a$, $8b$, $10c$, etc.

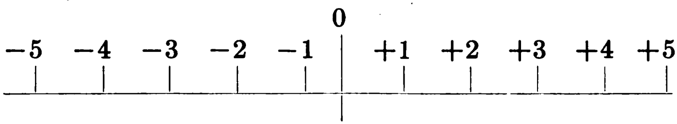
To add similar monomials find the algebraic sum of their coefficients and annex to this sum the letters common to the terms. Thus, to find the sum of $3a$, $5a$, $7a$, and $10a$. The sum of the coefficients is $3 + 5 + 7 + 10 = 25$. Hence, the sum of the monomials is $25a$.

Perform the following additions:

35.

(1) $2a$	(2) $9b$	(3) $10ab$	(4) $102by$
$5a$	$-7b$	$8ab$	$88by$
$7a$	$8b$	$-11ab$	$48by$
$9a$	$-5b$	$-4ab$	$-56by$
(5) $-444cx$	(6) $-356xy$	(7) $-32bx$	(8) $372xy$
$121cx$	$-472xy$	$149bx$	$256xy$
$-57cx$	$123xy$	$376bx$	$225xy$
$-42cx$	$97xy$	$-421bx$	$726xy$

58. Subtraction of Positive and Negative Numbers.—Subtracting a positive number is the same as adding an equal negative number. For, referring to the series again, we see that



the result of subtracting +1 from +3 is found by counting from +3 one unit in the negative direction, that is, in the direction opposite from that indicated by the sign + before 1, and is therefore +2.

The result of subtracting +1 from −3 is found by counting from −3 one unit in the negative direction, and is therefore −4.

Subtracting a negative number is the same as adding an equal positive number. Thus, the result of subtracting −1 from +3 is found by counting from +3 one unit in the positive direction, and is therefore +4.

The result of subtracting −1 from −3 is obtained by counting from −3 one unit in the positive direction, and is therefore −2.

To Subtract One Number from Another.—Change the sign of the number to be subtracted and proceed as in the rules for addition, for example:

EXAMPLES

36.

+14	−14	−14	−16
− 8	8	− 8	− 4
+22	−22	− 6	−12

Perform the following subtractions:

37.

(1)	(2)	(3)	(4)	(5)
− 9	17	15	−16	−11
18	21	−22	20	−17

(6) -18	(7) 19	(8) -19	(9) -4	(10) 18
-25	26	-18	14	$+18$

Subtraction of Monomials.—Change the sign of the coefficient of the term to be subtracted and proceed as in addition. Thus, to subtract $7a$ from $12a$ and $-9b$ from $15b$.

$$12a - 7a = 5a$$
$$15b - (-9b) = 15b + 9b = 24b$$

Perform the following subtractions:

38.

(1) $-4a$	(2) $7b$	(3) $16ab$	(4) $-9ac$
$6a$	$-8b$	$-21ab$	$17ac$
(5) $-22bc$	(6) $36ax$	(7) $72bx$	(8) $47ay$
$-25bc$	$56ax$	$-52bx$	$-471ay$

59. Multiplication of Monomials.—To multiply two or more monomials find the product of their numerical coefficients and if the literal factors (letters) are alike add the exponents of the letters.

Thus, $a^2 = aa$; and $a^3 = aaa$

$$a^2 \times a^3 = aa \times aaa = a^{2+3} = a^5$$
$$2a^2 \times 3a^3 = 2 \times aa \times 3 \times aaa = 6a^{2+3} = 6a_5$$

Law of Signs in Multiplication.—Like signs give $+$ and unlike signs give $-$. Thus, $a \times a = a^2$; $a \times (-a) = -a^2$; $2a^2 \times 3a^2 = 6a^4$; $2a^2 \times (-a^3) = -2a^5$.

A product of an odd number of negative factors gives a negative product and an even number of negative factors gives a positive product. Thus, $(-a) \times (-2a^2) \times (-3a^2) = -6a^5$; $(-a) \times (-2a^2) \times (+3a^2) = 6a^5$.

This will be seen more easily if we take the product of the first two factors in the first example. In this case we have $(-a) \times (-2a^2) = 2a^3$ which gives a positive product. Now if we complete the example by multiplying $2a^3$ by $-3a^2$ we have $-6a^5$.

Find the product of the following:

39.

(1) $3a$ $5a$	(2) $8a$ $7a$	(3) $-15b$ $-30b$
(4) $-20c$ $-18c$	(5) $-19ac$ $26ac$	(6) $32bc$ $-42bc$

40. $7ay \times 8ay \times (-15ay)$
41. $(-5xy) \times (-11xy) \times (-22xy)$
42. $(-16ab) \times (-18ab) \times ab$
43. $(-12ac) \times 14ac \times (-18ac)$

60. Division of Monomials.—To divide monomials divide their numerical coefficients and subtract the exponents of similar letters.

Thus, to divide $27a^3b^4$ by $3a^2b^2$ we have $27 \div 3 = 9$, $a^3 \div a^2 = a^{3-2} = a^1$, $b^4 \div b^2 = b^{4-2} = b^2$, or $27a^3b^4 \div 3a^2b^2 = 9a^{3-2}b^{4-2} = 9a^1b^2 = 9ab^2$.

Law of Signs in Division.—Like signs give + and unlike signs give −. Thus, $8a^3b^5 \div 4ab^2 = 2a^2b^3$, and $10a^4b^5 \div (-2a^2b^3) = -5a^2b^2$.

Work out the following problems:

44. Divide $54a^6$ by $9a^3$.
45. Divide $-36b^5$ by $-3a^2$.
46. Divide $32b^9$ by $2a^7$.
47. Divide $-72a^7c^5$ by $-8a^5c^3$.
48. Divide $-60a^9b^{10}$ by $4a^6b^5$.
49. Divide $27a^6b^8c^5$ by $-3a^5b^6c^4$.
50. Divide $-42x^2y^3$ by $-6xy$.
51. Divide $-81a^7b^3c^4$ by $3a^4b^2c$.
52. Divide $99a^9b^8c^7$ by $-11a^5b^4c^3$.
53. Divide $-64a^3x^5y$ by $4a^2b^4c^6$.

CHAPTER XII

SIMPLE EQUATIONS

61. Simple Equations.—An equation is a statement that two quantities are equal.

The first part of an equation, or that part to the left of the equality sign, is called the left-hand member of the equation, and the second part, or that part to the right of the equality sign, is called the right-hand member of the equation.

1. We may add the same thing to each side of an equation without destroying the equality. Thus, if $a = b$, we may add c to each, and the two sides will still be equal, $a + c = b + c$.

2. We may subtract the same thing from each side of an equation without destroying the equality. Thus, if $a = b$, then $a - c = b - c$.

3. We may multiply each side of an equation by the same thing without destroying the equality. Thus, if $a = b$, then $ac = bc$.

4. We may divide each side of an equation by the same thing without destroying the equality. Thus, if $a = b$, then $\frac{a}{c} = \frac{b}{c}$.

Example.—If out of 100 parts there are a cast parts and 30 forged parts, how many cast parts are there?

Solution.—Since there are 100 parts altogether, if 30 are forged parts and a are cast parts, then $a + 30 = 100$. Subtracting 30 from each side of the equation, we have

$$a = 100 - 30$$

or

$$a = 70 \text{ cast parts.}$$

PROBLEMS

1. Three machines, a, b, and c, cost \$1150, \$960, and \$$K$ respectively. What was the total cost of the machines?

2. A man buys 3 lb. of nails at 4 cts. per pound and 2 lb. of brads at a cts. per pound. If the nails and brads together cost 28 cts., what was the price per lb. of the brads?

3. Two pieces of machinery, a and b, cost \$310. If a cost \$50, what did b cost?

4. The total number of machines in a shop is 24, of which 6 are planers and a are lathes. Find the number of lathes.

5. If the total number of wheels on an engine is 10, and there are 3 drivers on a side, find the number of truck wheels on the engine. What is the type of the locomotive if there are no trailers?

6. If out of a total of 44 tracks in a roundhouse there are engines standing on x tracks, and there are still 22 tracks unused, how many engines are in the house?

In forming an algebraic equation we combine the terms according to the conditions stated in the problem and from the knowledge we have of the facts presented.

7. The length of a rectangular building is a ft., the width is 70 ft., and its area is 43,400 sq. ft. Find the value of a.

From the fact that the area of a rectangle is equal to the product of its two dimensions, we may form the algebraic equation by placing the area equal to the product of the length a ft. by the width 70 ft. Hence, $a \times 70 = 43{,}400$, which may be written $70a = 43{,}400$. Now, if 70 times a or $70a$ is equal to 43,400, a is equal to $1/70$ of 43,400 or 43,400 divided by 70, which may be written thus: $a = \frac{43{,}400}{70}$. Then, $a = 620$ ft. *Ans.* Hence 620 ft. is the length of the rectangle.

8. A rectangular shop building is a ft. long and 84 ft. wide. What is the length a, if the area is 13,440 sq. ft.?

9. A rectangular shop is a ft. long and b ft. wide. If $a = 48$ ft. and $b = 120$ ft., what is the area c?

10. The desired area of a new rectangular boiler shop is a sq. ft. Owing to the space available, the width is limited to b ft. What must be the length c, if $b = 72$ ft. and $a = 9648$ sq. ft.?

11. If the length of a rectangular shop is a ft. and the width 60 ft., and the floor space must be capable of accommodating 25 machines, each occupying an average of 300 sq. ft., find the length a.

12. The front section of an engine frame is required to have 48 sq. in. area, and the width is 4 in. and the depth is a in. Find the value of a.

13. The middle section of a rectangular shaped equalizer contains 24 sq. in. area, is 3 in. thick and a in. wide. Find the value of a.

62. Transposition of Terms.—It is convenient in solving a simple equation to bring the unknown letter or term, or unknown letters or terms, to one side of the equation, and all the other letters or terms to the other side. This process is called transposing the terms.

EXAMPLES

14. $a + b = c + d$. Solve for a. Subtracting "b" from each side of the equation we have

$$a + b - b = c + d - b$$

Cancelling the $+b$ and $-b$ we have

$$a = c + d - b$$

15. $c - d = x + y$. Solve for c. Adding "d" to each side of the equation we have

$$c - d + d = x + y + d$$

Cancelling $-d$ and $+d$ we have

$$c = x + y + d.$$

16. $\frac{a}{b} = cd$. Solve for a. Multiplying both sides of the equation by "b" we have

$$\frac{ab}{b} = bcd$$

or

$$a = bcd$$

17. $ab = cd$. Solve for b. Dividing each side of the equation by "a" we have

$$\frac{ab}{a} = \frac{cd}{a}$$

or

$$b = \frac{cd}{a}$$

From the foregoing rules, it is seen that any quantity may be transferred from one side of the equation to the other by performing the opposite operation on the side to which it is changed. That is, addition on one side is changed to subtraction on the other and vice versa; multiplication on one side is changed to division on the other and vice versa.

PROBLEMS

18. $a + b + c = x + y + z$. Solve for a.
19. $a + b - c = x - y - z$. Solve for b.
20. $x + y = ab$. Solve for x.
21. $x + y - z = \frac{a}{b} + c$. Solve for x.
22. $a + cd = b$. Solve for a.
23. $b + xy = c + d$. Solve for b.
24. $axy = bc$. Solve for x.
25. $abx = cy$. Solve for a.
26. $xyz = abc$. Solve for y.
27. $\frac{a}{b} = \frac{c}{d}$. Solve for a.
28. $\frac{x}{y} = \frac{b}{a}$. Solve for x.
29. $ax = c + d$. Solve for a.

From the rule that the speeds of two gears or pulleys are inversely proportional to their diameters, that is:

$$N : n = d : D \text{ or } ND = nd$$

where N and n are the respective speeds of the gears or pulleys in revolutions per min. and D and d are their diameters in inches or feet, etc., solve the following problems:

30. Given, the diameter D of a large pulley as 36 in., the speed N as 40 r.p.m., and the speed n of the small pulley as 130. Find the diameter of the small pulley.

31. What will be the speed of a 12-in. pulley which is belt connected to a 62-in. pulley making 115 r.p.m.?

The indicated horsepower of an engine is expressed by the following formula:

$$\text{h.p.} = \frac{PLAN}{33{,}000}$$

where h.p. = Horsepower.
P = Average cylinder pressure in lb. per sq. in.
A = Area of piston in sq. in.
L = Length of stroke in ft.
N = Strokes per min., or 2 × r.p.m. of drivers or fly-wheel for each cylinder.

32. Find the horsepower of an engine, one cylinder, if $P = 164$ lb. per sq. in.; $A = 452.39$ sq. in.; $N = 130$, and length of stroke is 26 in.

33. Rearrange the horsepower formula $\text{h.p.} = \frac{PLAN}{33{,}000}$ so as to give an expression for the area of the piston or A.

Ans. $A = \frac{\text{h.p.} \times 33{,}000}{PLN}$

34. Using the rule above, find the area required to develop 980 h.p. (one side) at 200 r.p.m. and 160 lb. pressure if the engine has a 24-in. stroke?

35. Rearrange the horsepower formula to give an expression for the number of strokes per min.

$$Ans.\ N = \frac{\text{h.p.} \times 33{,}000}{PLA}$$

36. Using the above rule for N, find the value of N when h.p. = 1250, P = 175 lb. per sq. in., and the cylinder is 24 in. × 28 in.

37. What is the expression for P in the horsepower formula?

38. If the value of h.p. per cylinder = 1000, $N = 160$, $A = 530.93$, and L = 2 ft. 6 in., what would be the steam pressure required?

39. Form the algebraic equation and solve for the diameter of the cylinder required to develop 1140 h.p. on one side with an average steam pressure of 150 lb. per sq. in., if the length of stroke is 2 ft. and the engine runs at a speed of 200 r.p.m. (Form the algebraic equation for the diameter d before doing any numerical substituting).

40. If a train travels E hr. at the rate of K miles per hr., find the distance traveled.

41. If a train travels M hr. at the rate of B miles per hr., and C hr. at the rate of D miles per hr., what is the total distance traveled?

42. A train travels C hr. without stopping, and then 2 hr. to complete a run of 10 hr. What is the value of C? (Work by a formula.)

43. What is the circumference in in. of a wheel D in. in diam.?

44. How many revolutions will a wheel D in. in diam. make in going 1 mile? What would this be for x miles?

45. What are the r.p.m. of a driver whose diameter is D in., when the locomotive is running at a speed of "X" m.p.h.?

$$Ans.\ \text{r.p.m.} = \frac{63{,}360X}{60 \times 3.14D}$$

46. Using the formula given above, what are the r.p.m. when D = 79 in. and X = 63?

47. Rearrange the formula in Prob. 45 to form an expression for D.

$$Ans.\ D = \frac{63{,}360X}{60 \times 3.14 \times \text{r.p.m.}}$$

48. What diameter of drivers is necessary to keep the r.p.m. at 300 when running at the rate of 70 m.p.h.? At 258 r.p.m. and 62 m.p.h.?

49. Rearrange the formula in Prob. 47 to form an expression for the m.p.h., (X).

$$\textit{Ans.}\ \text{m.p.h.} = \frac{\text{r.p.m.} \times 60 \times 3.14 \times D}{63{,}360}$$

50. What are the m.p.h. at 315 r.p.m. with an 80-in. diam. driver? At 250 r.p.m.? At 120 r.p.m.?

51. Referring to the horsepower formula and the formula for r.p.m.: Combine these formulæ to give an expression for the horsepower when the diameter of driver = "D" in. and the speed = "X" m.p.h., that is, for N put its value in terms of speed in m.p.h.

$$\textit{Ans.}\ \text{h.p.} = \frac{P \times L \times A \times 2 \times \text{m.p.h.}}{D \times .00297 \times 33{,}000} = \frac{P \times L \times A \times \text{m.p.h.}}{D \times 49}$$

52. Referring to the formula derived in the preceding question, what is the horsepower developed by a locomotive at a speed of 40 m.p.h., using 164 lb. as the effective pressure on the piston? Wheels = 80 in., cylinder = 24 in. × 26 in.

53. What would be the horsepower of a locomotive running at 60 m.p.h. if we used 145 lb. per sq. in. as the effective pressure in place of 164 lb., the cylinders and drivers being the same as in the preceding question?

54. What is the horsepower developed by a locomotive running at a speed of 10 m.p.h., using $\frac{4}{5}$ boiler pressure as the effective pressure? Wheels = 80 in., cylinder = 22 in. × 26 in., boiler pressure = 205 lb. per sq. in. What is the horsepower at 30 m.p.h.?

55. What is the horsepower developed by a freight locomotive at 10 m.p.h.? Drivers = 62 in., cylinders = 24 in. × 28 in., boiler pressure = 205 lb., $P = \frac{4}{5}$ boiler pressure.

The amount of electric current which will flow in a circuit of given resistance under a given pressure is found from the following rule, known as Ohm's Law:

$$\text{Current} = \frac{\text{voltage}}{\text{resistance}} \text{ or } C = \frac{V}{R}$$

56. What is the voltage on a circuit having a resistance of 4 ohms and through which a current of 25 amperes flows?

57. A current of 32 amperes flows through a circuit under a pressure of 110 volts. What is the resistance of the circuit?

The law of gravity for falling objects is expressed by the formula

$$h = \frac{gt^2}{2}$$

where h = Height in ft. from which object falls.
g = Acceleration = 32.16 ft. per sec. each sec.
t = Time in sec.

58. How long will it take an object to fall to the earth from a height of 350 ft.?

59. The height of a building is 720 ft. How long would it take a baseball to hit the ground if it were dropped from the top of the building?

The rule for finding the velocity in ft. per min. of the rim of a wheel or pulley is expressed by the formula

$$V = \frac{2\pi RN}{12}$$

where V = Velocity in ft. per min.
R = Radius of wheels in in.
N = R.p.m.

60. Find the velocity of the rim of a fly-wheel 120 in. in diam. making 90 r.p.m.

61. The rim of a pulley has a velocity of 4021 ft. per min. What is its diameter if it makes 320 r.p.m.?

62. Good practice says that the velocity of the rim of a fly-wheel should not exceed 5280 ft. per min. What then should be the greatest allowable speed in r.p.m. of a fly-wheel 96 in. in diam.?

The rule for finding the width of a single belt to transmit a given horsepower is as follows:

$$W = \frac{33{,}000 \times H}{P \times S}$$

in which W is the width of the belt in in.; H, the horsepower transmitted; P, the allowable pull per in. of width of belt; and S, the speed of the belt in ft. per min. For example, to find the width of a belt required to transmit 13 h.p., with an allowable pull on belt per in. of width of 30 lb. and the belt running at 3000 ft. per min., we have

$$W = \frac{33{,}000 \times 13}{30 \times 3000} \text{ or } 4.76 \text{ in.}$$

We would therefore use a 5-in. belt as this is the next larger size.

To find the horsepower which a given size belt will transmit at a given speed and with a given allowable pull per in. of width of belt, we rearrange the rule above to give an expression for H in terms of P, S and W as follows:

$$H = \frac{W \times P \times S}{33{,}000}$$

and using this rule we find that a belt 8 in. wide, running at 3500 ft. per min. with an allowable pull of 40 lb. per in. of width, will transmit safely 33.9 or practically 34 h.p., found as follows:

$$H = \frac{8 \times 40 \times 3500}{33{,}000} \text{ or } 33.9$$

The above rule worked out for P is

$$P = \frac{33{,}000 \times H}{W \times S}$$

and for S is

$$S = \frac{33{,}000 \times H}{W \times P}$$

The rule for finding the electrical horsepower of a direct-current generator is to multiply the voltage at the generator terminals by the current output of the generator and divide the product by 746, or written as an equation we have

$$\text{Horsepower} = \frac{\text{volts} \times \text{amperes current}}{746}$$

From this same rule,

$$\text{Volts} = \frac{\text{horsepower} \times 746}{\text{amperes current}}$$

and also

$$\text{Amperes current} = \frac{\text{horsepower} \times 746}{\text{volts}}$$

Example.—A generator delivers 25 amperes at 110 volts to a bank of lamps. What is the horsepower output of the generator?

$$\text{Horsepower} = \frac{110 \times 25}{746} = 3.68$$

Example.—A direct-current motor takes in 50 h.p. at a voltage of 220. What is its current intake?

$$\text{Amperes current} = \frac{50 \times 746}{220} = 169.5$$

PROBLEMS

63. What horsepower is delivered by a direct-current generator which supplies 120 amperes at a voltage of 110 volts?

64. In figuring up the lighting for a shop it is found that the total current required is 42 amperes, and if 220-volt lamps are to be used, what horsepower generator is required?

65. A motor driving a lathe takes 22 amperes direct current at 110 volts. What horsepower is it taking?

66. A 10-h.p. motor running at rated load receives power from a 220-volt main. How much current does it take from the main?

To find the brake horsepower of an engine an arrangement such as is shown in Fig. 112 is used, and the formula is:

$$\text{b.h.p.} = \frac{2\pi PRN}{33{,}000}$$

when b.h.p. = Brake horsepower.
R = Length of arm in ft.
N = R.p.m.
P = Reading of scale beam.

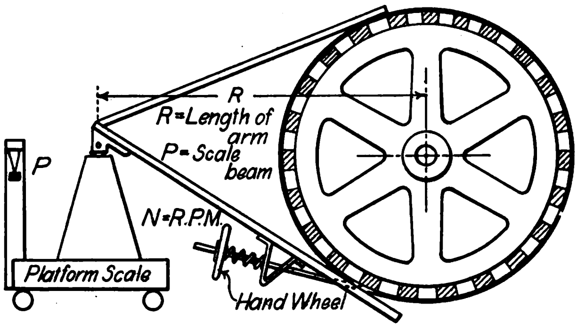


FIG. 112.

67. The pulley of an engine makes 250 r.p.m.; the brake arm is 48 in. long and the weight at end of arm is 180 lb. What is the b.h.p. of the engine?

68. In a test an engine made 120 r.p.m., and the brake arm was 54 in. long. What was the weight on the scales? The brake horsepower was 30.

CHAPTER XIII

THE BINOMIAL

63. Definitions and Miscellaneous Problems.—A *binomial* is an expression containing two terms, such as $a + 8$, $3a + 2d$, $2ab - c$, $4 + 2a$, etc.

Addition and Subtraction of Binomials.—To add two or more binomials put like terms under like terms and add. That is, we take the like terms, which are monomials, and proceed as in the addition of monomials.

For example, if we wish to add $a + b$, $2a + b$ and $3a + 2b$ we have $a + 2a + 3a$ or $6a$ and $b + b + 2b$ or $4b$ and $(a + b) + (2a + b) + (3a + 2b) = 6a + 4b$, or proceeding in the usual manner, we have

$$\begin{array}{r} a + \ \ b \\ 2a + \ \ b \\ \underline{3a + 2b} \\ 6a + 4b \end{array}$$

To subtract one binomial from another we subtract like terms from like terms, the procedure being the same as with monomials.

Example.—Suppose we wish to subtract $2ab + c$ from $3ab + 3c$. Then $3ab - 2ab = ab$ and $3c - c = 2c$ and the difference is $ab + 2c$, or proceeding in the usual manner:

$$\begin{array}{r} 3ab + 3c \\ \underline{2ab + \ \ c} \\ ab + 2c \end{array}$$

PROBLEMS

Add the following binomials:

1. $(a + b), (a + 2b), (3a + 4b)$.
2. $(a + 2c), (2a + c), (3a + 5c), (5a + 4c)$.
3. $(c + 2d), (4c - 2d), (3c \dot{-} d)$.
4. $(2ab + c), (ab - 2c), (-3ab + 4c)$.
5. $\frac{1}{2}(a + b), \frac{1}{2}(a - b)$.

Subtract the following binomials.

6. $(3x + 2y) - (2x + y)$.
7. $(7a - 3b) - (5a + 3b)$.
8. $(4ab + 5bc) - (3ab - 6bc)$.
9. $(4a + 7xy) - (6a - 5xy)$.
10. $\frac{1}{2}(a + b) - \frac{1}{2}(a - b)$.

Multiplication of a Binomial by a Monomial.—To multiply a binomial by a monomial, we multiply each term of the binomial by the monomial and prefix to it the proper sign.

EXAMPLES

11. $2a(3a + 2b) = ?$

$2a \times 3a = 6a^2$ and $2a \times 2b = 4ab$.

or $2a(3a + 2b) = 6a^2 + 4ab$.

12. $4c\,(3c - 5d) = ?$

$4c \times 3c = 12c^2$ and $4c \times (-5d) = -20cd$.

or $4c(3c - 5d) = 12c^2 - 20cd$.

PROBLEMS FOR PRACTICE

13. $3a(a + b) = ?$
14. $a(2b + c) = ?$
15. $b(a - c) = ?$
16. $c\left(\frac{a + b}{2}\right) = ?$
17. $2a\left(\frac{4b - 2c}{c}\right) = ?$
18. $2ab\left(\frac{2x - y}{4}\right) = ?$

The sum of the squares of the legs of a right triangle is equal to the square of the hypothenuse. Let a and b, Fig. 113, be the lengths of the two legs and c the length of the hypothenuse of a right triangle, then

$$a^2 + b^2 = c^2$$

and

$$c = \sqrt{a^2 + b^2}$$
$$a = \sqrt{c^2 - b^2}$$
$$b = \sqrt{c^2 - a^2}$$

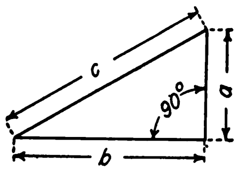


FIG. 113.

PROBLEMS

19. If a is the length of one side of a right triangle, 12 in. the length of the other side, and 25 in. the length of the hypothenuse, what is the length of the side a?

20. What is the length of the hypothenuse of a right triangle whose legs are 8 in. and 14 in. respectively?

21. One leg of a right triangle is 16 in. long and the hypothenuse is 20 in. long. Find the length of the other leg.

The area of a trapezoid is expressed by the following formula:

$$A = h\left(\frac{a + b}{2}\right)$$

When A = Area.
a = Length of upper base.
b = Length of lower base.
h = Vertical height of trapezoid.

22. What is the area of a trapezoid whose bases are 12 in. and 16 in. and whose altitude is 10 in.?

23. Find the altitude of a trapezoid having bases 24 in. and 32 in. long and an area of 560 sq. in.?

24. A trapezoid has an area of 368 sq. in. Its altitude is 16 in. and its lower base is 26 in. long. What is the length of its upper base?

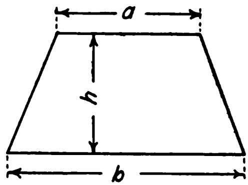


Fig. 114.

To find the proper size of drill to use for a given size United States Standard tap, the following rule is used.

$$D = T - \frac{1.3}{N}$$

in which, D = Diameter of drill.
T = Outside diameter of tap.
N = Number of threads per in.

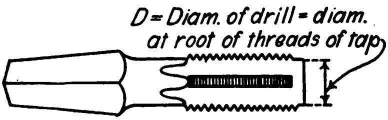


Fig. 115.

Example.—Find the size of drill for a 1-in. tap, and give the size in the nearest thirty-second of an inch.

Solution.—$D = 1 - \frac{1.3}{8} = 1 - .1625$ or $.8375$ in.

The "8" is used in the denominator of the fraction in the rule since for a 1-in. bolt there are 8 threads per in.

To change .8375 in. to the nearest thirty-second, we multiply it by 32: $.8375 \times 32 = 27+$. Therefore the size drill required is $\frac{27}{32}$ in.

PROBLEMS

25. Find the size of drill for a 1½-in. tap, there being 6 threads per in.

26. Find the size of drill required for 2-in. tap having 4½ threads to the inch.

To find the diameter of a circle when only a portion of the circumference is given. In Fig. 116 we have given the arc *acb*. To find the diameter of the circle of which this arc is a portion, we draw in the chord "*C*" and the line "*S*" at right angles to the chord at its middle point.

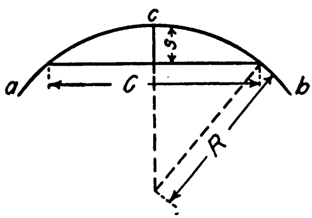


FIG. 116.

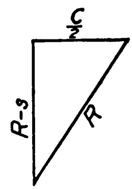


FIG. 117.

The diameter of the circle then equals $\dfrac{\left(\dfrac{C}{2}\right)^2 + S^2}{S}$.

The line "*S*" is the height of the segment formed by the chord "*C*" and the arc of the circle.

Proof of rule for finding diameter of a given arc.

$$R^2 = (R - S)^2 + \left(\frac{C}{2}\right)^2$$

$$R^2 = R^2 - 2RS + S^2 + \left(\frac{C}{2}\right)^2$$

or $$2RS = \left(\frac{C}{2}\right)^2 + S^2$$

$$\text{and} \qquad 2R = \frac{\left(\frac{C}{2}\right)^2 + S^2}{S}$$

$$\text{that is} \qquad D = \frac{\left(\frac{C}{2}\right)^2 + S^2}{S}$$

Stated in words this rule is: To find the diameter of a given arc draw in a chord and find the height of the

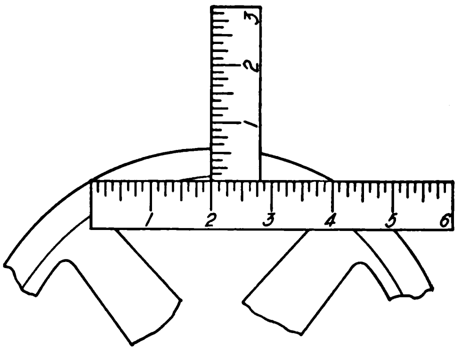


Fig. 118.

segment thus formed. The diameter then equals the square of one-half the length of chord plus the square of the height of segment all divided by the height of segment. To apply this rule in finding the diameter of the broken pulley shown in Fig. 118, the length of the chord measures 4 in. and the height of segment $\frac{9}{16}$ in.

Using the rule the diameter

$$D = \frac{\left(\frac{4}{2}\right)^2 + \left(\frac{9}{16}\right)^2}{\frac{9}{16}} \quad \text{or} \quad D = \frac{4 + \frac{81}{256}}{\frac{9}{16}}$$

Working out the fraction we have

$$D = \frac{4\frac{81}{256}}{\frac{9}{16}} = \frac{1105}{256} \times \frac{16}{9} \text{ or } \frac{1105}{144}$$

This fraction simplified comes out $7\frac{97}{144}$ or 7.67 in.

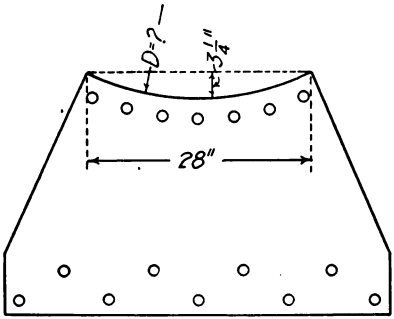


Fig. 119.

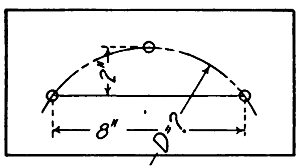


Fig. 120.

27. Figure 119 shows a belly brace sheet. From the dimensions given find the diameter of the curved edge of the sheet.

28. Find the diameter of the circle which would pass through the centers of the three holes in the steel plate shown in Fig. 120.

CHAPTER XIV

THE USE OF TABLES AND CURVES

64. Tables and Curves Used in Solving Problems.—It is often convenient to use a table to show the value of quantities compared with one another, as, for example, the number of threads per in. and the corresponding size

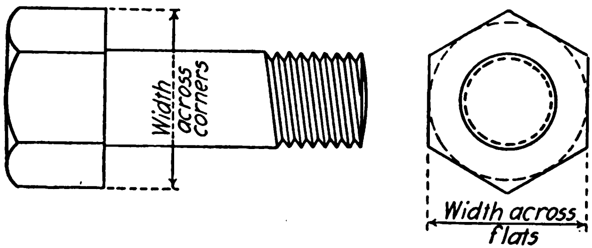


FIG. 121.

of bolt heads for different diameter bolts. Such a table is as follows:

Threads per in.	Diameter of bolt	Width across flats	Width across corners
20	.25	.50	$\frac{37}{64}$
13	.50	.875	1
10	.75	1.25	$1\frac{7}{16}$
8	1.00	1.625	$1\frac{7}{8}$

etc., etc.

These values have been calculated from rules and save the work of computation each time if we have the table for reference. In some cases tables are put on machines, such as lathes, to show the proper gears to use for any given number of threads per in. Tables are also provided showing the proper speed at which to run lathes for different sizes of work being turned down and for different kinds of material. In a broker's office tables are used to calculate interest and to show the change in price of different stocks during a given period.

Even more convenient than tables are curves drawn on "squared" or "cross-sectioned" paper to convenient scales and showing at a glance the relation between the quantities considered. For example, the following table

Revolutions per min.	Miles per hr.
0	0
75	17.8
150	35.7
200	47.6
250	59.5

gives the revolutions per min. of an 80-in. diam. locomotive driver and the corresponding speed of the locomotive in miles per hr. The same figures are shown by the straight-line diagram, Fig. 122. The values are calculated from the rule,

$$\text{Speed in miles per hr.} = \frac{D \times 3.14 \times \text{r.p.m.} \times 60}{12 \times 5280}$$

in which D is the diameter of the driver in in. and r.p.m. is the number of revolutions of the driver per min.

The paper used is called "squared" paper, "cross-sectioned" paper, "coordinate" paper or "plotting" paper. It is laid off in squares as shown by the equidistant horizontal and vertical lines.

On this paper we first plot or lay off points from figures in the table from which we are working, these figures having previously been obtained from calculation using a formula or rule, or else obtained from observation or experiment. The scales chosen are such as suit the data we wish to plot. In this case the speeds go up to 70 miles

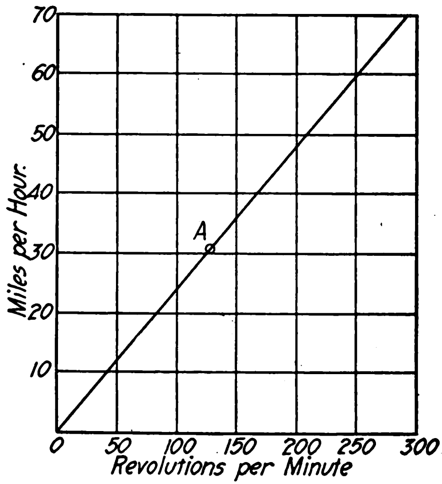


Fig. 122.

per hr., which is about as high a speed as it is safe for a locomotive to run. The corresponding highest number of revolutions per min. is approximately 295. In order therefore to get these values on the sheet we use the scales as shown. The small circles show the location of the points plotted from the figures in the table. These points lie along a straight line from which we can find the speed for any given number of revolutions per min., or vice versa, within the limits given on the paper. Using the line plotted we see that for 105 r.p.m. the corresponding speed is 25 miles per hr. Again for 50 miles per hr. the corresponding revolutions per min. are 210.

The general name "curve" is given to a line drawn through points plotted from observational or calculated data, although the "curve" may in some cases be as in this one a straight line. Plotted data is often called a "graph," "chart" or "diagram."

The quantities representing distances that are plotted horizontally and vertically are called the "coordinates"

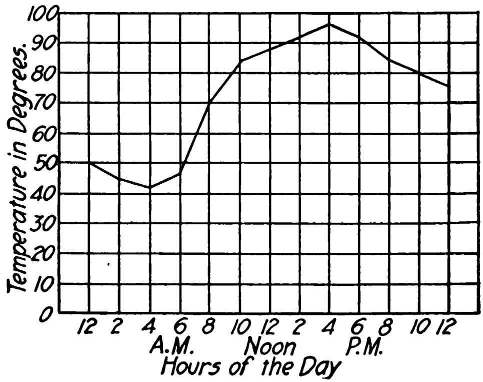


Fig. 123.

of the point in question, the former the "abscissa" and the latter the "ordinate" of the point.

For example, the coordinates of the point "*A*" in the diagram are abscissa, 130 r.p.m.; ordinate, 31 miles per hr. Laying out a curve from the figures given is called "plotting" the curve.

The following table and plot corresponding, Fig. 123, show the change in temperature during a day in summer:

Hour of the day	Temperature in degrees
12	50
2	44
4	42
6 A.M.	46
8	70
10	84
12 Noon	88
2	92
4	96
6 P.M.	92
8	84
10	80
12	78

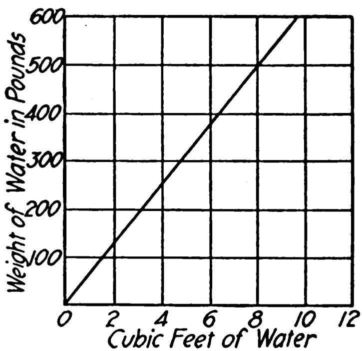


FIG. 124.

The horizontal distances represent the hours of the day and the vertical distances represent the corresponding temperature in degrees Fahrenheit. As we should expect, the temperature at noon was considerably higher than in the morning, and it may also be seen that the evening was hot. Such charts are made for each day of the year. They show the temperature variations at a glance and are much easier read and more easily understood than a table.

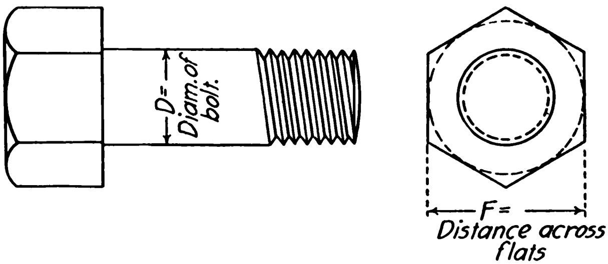


FIG. 125.

The chart, Fig. 124, shows the relation between the weight of water in pounds and cubic feet of water. Using this chart, we can find the weight of any number of cubic

Diameter of bolt	Width across flats
.5	.875
1.0	1.625
1.5	2.375
2.0	3.125
2.5	3.875
3.0	4.625

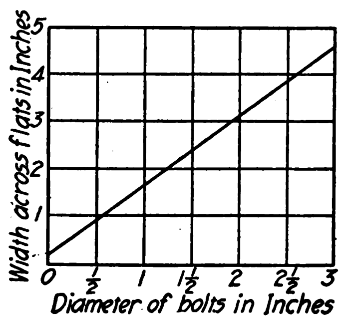


FIG. 126.

feet of water within the limits of the diagram. Also we can find how many cubic feet of water are required for a given weight. The table and chart are calculated from the following rule: Weight of water in pounds = cubic feet of water × 62.5, since 1 cu. ft. of water weighs 62.5 lb.

The rule for finding the distance across flats of a standard hexagonal bolt head is as follows:

$$F = 1\tfrac{1}{2}D + \tfrac{1}{8}$$

in which D is the diameter of the bolt.

From calculations using this rule the table and chart, Fig. 126, are made. Using this chart, find as closely as you can the distance across flats of a 1¼-in. bolt. Then using the rule given above, calculate "F" and compare it with your reading.

PROBLEMS

1. Using the plot showing the miles per hour and the corresponding number of revolutions per min., find the speed for 120 revolutions per min. (r.p.m.) and also for 285 r.p.m. Find also on this chart the r.p.m. corresponding to 49 miles per hr. (m.p.h.); also the r.p.m. for 64 m.p.h.

2. Make a chart to convenient scale, plotting the numbers 1, 4, 7, 11, 14, 18 and 21 as horizontal distances or "abscissæ" and their square roots as corresponding vertical distances or "ordinates."

From this chart find the square roots of the following numbers by connecting the points plotted with a smooth curve and reading from this curve. Numbers: 5, 12, 15 and 20. Compute the square roots of these numbers and see how they compare with the values read from the chart.

3. Make a chart showing the weight of steel as ordinates and cubic inches of steel as abscissæ, using the rule: Weight of steel in lb. = cubic inches of steel × .283.

4. A man's yearly salary for different ages is given by the following figures:

Age	Salary
20	$600
30	1000
40	1800
50	3200
54	3800

Make a chart showing these figures and from the curve estimate what his salary will be when he is 60 yr. old if his increase is at the same rate as shown by the curve.

CHAPTER XV

CUBE ROOT

65. Definitions and Problems.—The cube root of a number is a quantity which when cubed or raised to the third (3rd) power gives that number. For example: the cube root of 8 is 2 because 2^3 or $2 \times 2 \times 2$ is 8. Also the cube root of 27 is 3 because 3^3 or $3 \times 3 \times 3$ is 27. The sign of the cube root is $\sqrt[3]{\quad}$. Thus we have $\sqrt[3]{27} = 3$, $\sqrt[3]{125} = 5$.

The cube root of a number may be considered as the side of a cube the contents of which represents the number. Thus, 10^3 or $10 \times 10 \times 10$ is 1000 and a cube 10 in. on a side has a volume of 1000 cu. in.

Therefore the length of one side of a cube equals the cube root of the number representing the volume of the cube. There are a number of ways of finding the cube root of a number: (1) By actual calculation, (2) by use of a table, (3) by the use of logarithms which we will take up later, and (4) by the use of some computing device like the slide rule. In this book may be found a table of cubes and cube roots of numbers, and from this table the answers required in cube root may be found very closely. In some cases it is better to work out cube roots than rely entirely on a handbook or table.

In working out the cube root of any number first find the number of figures there are in the root. This is done by pointing off the number into groups of three figures each, beginning with the figure to the extreme right for a whole number or at the decimal point in a decimal or mixed number. The number of figures in the root is the same as the number of groups in the number whose cube root we are finding. Thus the cube root of 8′741′816 has three figures because it contains three groups. The cube root of 19′683 contains two figures since there are two groups. (The extreme left-hand group may have one, two, or three figures in it.)

Example.—Find the cube root of 10,077,696.

First point off the number into groups of three figures each. This gives three groups.

		10′077′696(216 *Ans.*
$2^3=2\times2\times2=$		8
$2^2\times300=4\times300=$	1200	2077
$2\times1\times30=$	60	
$1^2=1\times1=$	1	
	1261	1261
$21^2\times300=21\times21\times300=$	132300	816696
$21\times6\times30=$	3780	
$6^2=6\times6=$	36	
	136116	816696

The largest number whose cube we know will go in 10 is 2. This is the first figure in the answer. Next we subtract the cube of 2 from the first group and with the remainder bring down the next group 077, giving 2077. Next we square the root already found and multiply it by 300. See how many times this goes in 2077. As 1200 is our trial divisor and is contained into 2077 but once, we then take 1 as our next figure to go in the answer. Then we take the product of these two numbers found in the answer (which is 2 × 1 or 2) and multiply it by

30, which gives $2 \times 1 \times 30$ or 60. Then square the last root figure in the answer, which in this case is 1, and then add all these divisors together and we have 1261 as the total divisor. Multiply this divisor by the root figure, 1, and subtract this from 2077 and we have 816 for a remainder. Bring down the next group 696 and proceed as before, and after we have added all the divisors we multiply the total or 136116 by our last figure in the answer, 6, and the operation is then completed as there is no remainder.

If it is desired to find the cube root of a number composed of a whole number and a decimal, begin at the decimal point and point off groups to the right and left. Then find the root as before. Sometimes in the case of a decimal the first group is composed of ciphers, as .000′015′625. In this case the first figure of the root is a cipher. The cube root of .000015625 is found as follows:

$$
\begin{array}{rr}
 & .000'015'625(\underline{.025}\ \textit{Ans.} \\
 & \underline{8} \\
2^2 \times 300 = 2 \times 2 \times 300 \text{ or } 1200 & 7625 \\
2 \times 5 \times 30 = 300 & \\
5^2 = 5 \times \underline{5 =} \quad 25 & \\
1525 & \underline{7625}
\end{array}
$$

PROBLEMS

Find the cube root of the following numbers:

	Number	Answer
1.	2	1.259921
2.	7	1.9129312
3.	274,625	65
4.	1,030,301	101

	Number	Answer
5.	5,545,233	177
6.	25,934,336	296
7.	69,934.528	41.2
8.	172,808,693	557
9.	4,057,719,875	1595
10.	3,996,969.003	158.7

The answers to the above are given to aid in understanding the method used. Work out each example for yourself, step by step, as explained in the examples given.

Suppose it is desired to find the dimensions of a cubical coal bin to hold 1000 cu. ft. In this case the length, breadth, and depth of the bin each equal the cube root of 1000 cu. ft. or 10 ft.

If it is desired to find the diameter of a sphere to contain a certain volume, divide the volume by $\frac{\pi}{6}$, that is $\frac{3.1416}{6}$ or .5236, and take the cube root of the result. Many other uses of cube root arise in practice and it is well to have a clear understanding of how to find the cube root of any number, large or small, whether a whole number, a fraction or a mixed number.

PROBLEMS

11. It is desired to find the dimensions for a cubical shaped coal bin to hold 1000 tons. If each ton occupies 37 cu. ft., find the length of each side of the bin.

12. One hundred steel ball bearings weigh 1 lb. If each cubic inch of metal in the bearings weighs .28 lb., find the diameter or size of the bearings.

Rule.—Volume of a sphere $= \frac{3.1416}{6}$ (diameter)^3

or the diameter $= \sqrt[3]{\frac{\text{volume} \times 6}{3.1416}}$.

13. It is desired to construct a cylindrical tank to hold 50 cu. ft. so that the amount of material used will be the least possible.

Rule.—The material required is the least when the height and diameter of the base are equal. Therefore referring to Fig. 127, if the base and height are each equal to d ft. the volume equals $d \times d \times .785 \times d = 50$ cu. ft., or $.785d^3 = 50$ and $d = \sqrt[3]{\frac{50}{.785}}$. Find "$d$."

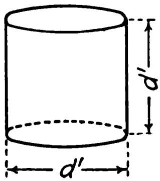


Fig. 127.

14. How many times heavier are 1000 steel ball bearings ¼ in. in diam. than the same number ⅛ in. in diam.? The steel weighs .28 lb. per cu. in.

15. What is the weight of a lead ball 4 in. in diam. if lead is 11.4 times as heavy as water? (Water weighs 62.5 lb. per cu. ft.)

16. How much space is left between the surface of a 2-in. sphere and the inside surface of a cubical box which the sphere just touches?

17. A test is made to find the diameter of ball bearings by finding the amount of water they displace. When 100 of the balls are put in a cylindrical vessel 4 in. in diam. the water rises 4.17 in. What is the diameter of the bearings?

CHAPTER XVI

MENSURATION OF PRISMS, PYRAMIDS AND CONES

66. Definitions. Prisms.—In shop work it is often necessary to calculate the surface or volume and weight of objects which are either solids or containers for solids, liquids, or gases.

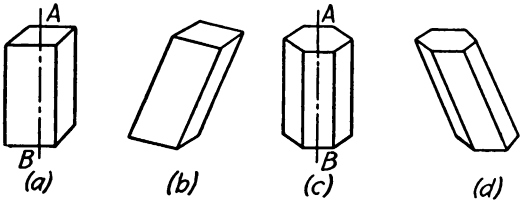


FIG. 128.

Solids, as shown in Fig. 128, whose ends are equal polygons (plane figures) in parallel planes and whose sides are parallelograms (that is, four-sided plane figures whose opposite sides are equal and parallel) are called "Prisms."

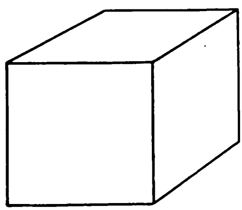


FIG. 129.

Figures (*a*) and (*c*) are right prisms as the center lines *AB* are perpendicular to the base of the solids. Fig. (*a*) shows a square prism, since its base is a square; and Fig. (*c*) is a hexagonal prism, the base being a hexagon.

Figure 129 shows a *cube* or solid, every face of which is a *square*. *The entire area of a prism* is equal to the sum of the areas of all its faces.

The *volume or contents of a prism* is found by multiplying the area of its base by its altitude or vertical distance between the upper and lower bases.

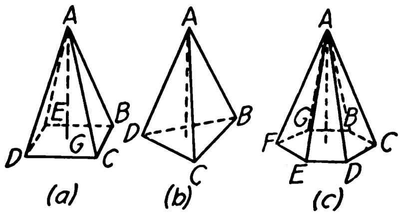


Fig. 130.

67. Pyramids and Cones.—A *pyramid* is a solid (Fig. 130(*a*)) whose base is a polygon of three or more sides and whose sides are *triangles* uniting at a common point *A* (Fig. 130) called the "apex."

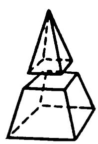


Fig. 131.

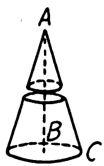


Fig. 132.

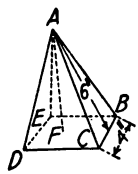


Fig. 133.

A "regular pyramid" has a "regular polygon" for its base and all its sides have the same inclination to the base. Fig. 130(*a*) shows a *square* pyramid since its base is a square. Fig. 130(*b*) shows a *triangular* pyramid, the base being a triangle; Fig. 130(*c*) shows a *hexagonal* pyramid.

Figure 131 shows a *pyramid* cut by a *plane* parallel to the base, forming two parts. The lower part is called a *frustum* of the pyramid.

A *cone* cut in the same way as in Fig. 132 forms the *frustum* of a cone.

The entire area of a *pyramid* is equal to the sum of the areas of all its faces. In Fig. 133 the *square* pyramid shown has a base 4 in. on each side and the height of each triangle forming the sides is 6 in. The total area equals area of base plus four times area of one side, or total area equals

$$4 \times 4 + \frac{4 \times 4 \times 6}{2} = 16 + 48 = 64 \text{ sq. in.}$$

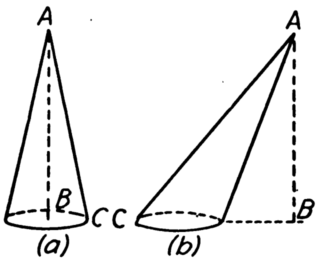


FIG. 134.

The *volume* or *contents* of a pyramid is found by multiplying the area of its base by one-third its altitude or vertical distance between the apex (point *A*, Fig. 133) and the base. In Fig. 133 the volume equals area of base × ⅓ the height or

$$4 \times 4 \times \frac{5.68}{3} = 30.3 \text{ cu. in.}$$

Figure 134 shows the *cone* or solid whose base is a *circle* and whose surface above the base tapers uniformly to a point A called the *apex*. In each figure, (a) and (b), the distance AB is the altitude or vertical height and AC is the slant height. The *convex area* or entire area of a cone, *except* that of the base, equals the *circumference of the base* multiplied by *one-half* the *slant height*.

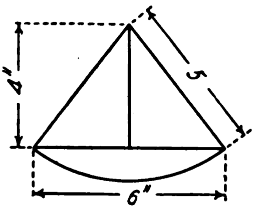


FIG. 135.

The entire area of a cone equals the *convex* area plus the area of the base. In Fig. 135 the cone has a base 6 in. in diam. and an altitude of 4 in. and slant height of 5 in. Its *convex area* equals $\pi 6 \times 5/2$ or 47.1 sq. in. The *entire area* equals the convex area plus the area of the base or

$$\textit{Entire area} = 47.1 + \pi(3)^2 = 47.1 + 28.3$$
$$= 75.4 \text{ sq. in.}$$

The *volume* or *contents* of a cone equals the area of the base multiplied by *one-third* the altitude. The volume of the cone shown in Fig. 135 equals $6 \times 6 \times .785 \times 4/3$ or 37.7 cu. in.

PROBLEMS

1. Find the weight of a cast-iron square pyramid with base 3 in. on a side and height of 9 in.

2. Find the weight of a cast-iron cone with a base 3 in. in diam. and height of 6 in.

3. If 1 gal. of paint costing $1.80 covers 300 sq. ft., find the cost of painting a conical shaped roof 31.4 ft. around the base and with a slant height of 8 ft.

4. What is the weight of a cast-iron pyramid (square) with a base 6 in. on a side and a height of 10 in. if the cast iron weighs .26 lb. per cu. in.?

5. Find the total surface of a hexagonal pyramid with a base 2 in. on a side and a slant height of 12 in. Find the volume of the same pyramid and its weight if made of cast iron weighing .26 lb. per cu. in.

6. Find the weight of a pointed steel block as per Fig. 136 if the steel weighs 483 lb. per cu. ft.

7. Find the volume and weight of the crank pin forging shown in Fig. 137 if the steel weighs .28 lb. per cu. in.

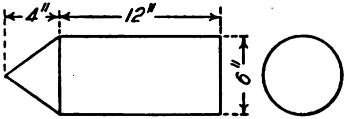


FIG. 136.

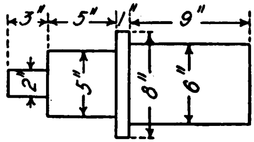


FIG. 137.

68. Frustums of Pyramids and Cones.—In Figs. 138 (*a*) and (*b*), showing frustums of a square pyramid and of a cone, the planes *ABCD* form the *upper base* and the planes *EFGH* the *lower base*. The altitude of the frustum is the *perpendicular* distance between the bases.

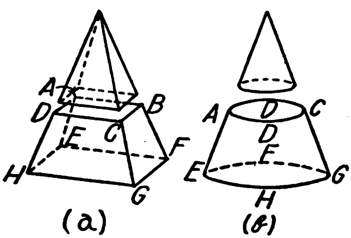


FIG. 138.

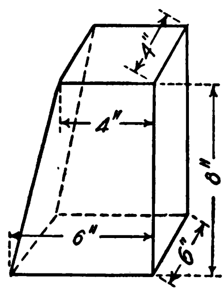


FIG. 139.

To find the *convex area* of the frustum of a pyramid or cone (that is, the entire area *except* that of the upper and lower bases), find the distance around (or the perimeter of) the upper and lower bases, add these together, divide the sum by 2 and multiply by the slant height of the frustum. In Fig. 139 the *perimeter* of the upper base equals 16 in., of the lower base 24 in., the sum being equal to 40 in. One-half the sum equals 20 in. and the slant height AB is 8 in. The *convex area* = 20 × 8 = 160 sq. in. The *entire area equals* the *convex area* plus the areas of the upper and lower bases or, in Fig. 139, *total area* = 160 + 4 × 4 + 6 ×6 = 160 + 16 + 36 or 212 sq. in. Using the same rule for the frustum of a cone, we find *one-half the sum of the circumferences* of the *upper* and *lower bases* and multiply this by the slant height of the frustum, giving the *convex area.* The *total area* then equals the convex area plus the areas of the upper and lower bases.

If the diameters of the upper and lower bases of the frustum of a cone are respectively 4 in. and 6 in. and the slant height is 8 in., the *convex area* equals

$$\frac{\text{Circ. of upper base} + \text{circ. of lower base}}{2} \times \text{slant height}$$

or

$$\frac{\pi 4 + \pi 6}{2} \times 8 = \frac{10\pi}{2} \times 8 = \pi 40 = 125.6 \text{ sq. in.}$$

The total area then equals *convex area* plus areas of upper and lower bases, or $125.6 + \pi 4 + \pi 9 = 125.6 + 40.8 = 166.4$ sq. in.

The rule for finding the *volume* of the frustum of a *pyramid* or *cone* is most easily expressed in symbols as follows:

$$V = (A + B + \sqrt{A \times B})\,\frac{H}{3}$$

in which V is the volume of the frustum, A the area of its upper base, B the area of its lower base, and H the altitude or vertical distance between the bases.

The frustum of the *square pyramid* represented in Fig.

140 has an upper base 4 in. on a side, the lower base is 6 in. on a side, and the altitude is 9 in. The *volume* then is $(4 \times 4 + 6 \times 6 + \sqrt{4 \times 4 \times 6 \times 6})\ 9/3$, or the volume equals $(16 + 36 + \sqrt{16 \times 36})9/3$ or $(16 + 36 + 24)3 = 76 \times 3$ or 228 cu. in.

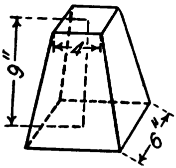


FIG. 140.

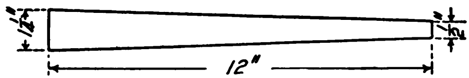


FIG. 141.

69. Tapers and Taper Turning.—A taper is usually expressed as so many inches or part of an inch per ft. of length of the work. As for example, a taper 3/4 in. per ft. means that if the work is 1 ft. long as in Fig. 141 the diameter of the small end is 3/4 in. smaller than the diameter at the large end. If the work was 1½ ft. long with a taper of 3/4 in. per ft. the difference between the

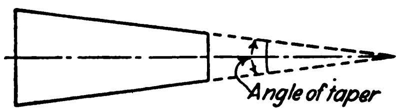


FIG. 142.

large and small diameters would be $1\frac{1}{2} \times \frac{3}{4}$ or $1\frac{1}{8}$ in. A taper of 3/4 in. per ft. means a taper of 1/12 of 3/4 or 3/48 or 1/16 in. per in. of length.

If the angle of taper is given it means the angle formed by the two sides of the work, as in Fig. 142. The angle between the center line and side of the work is one-half of the total angle of taper.

The *amount of taper* depends not only on the diameter at each end of the taper, but also on the length of the taper. In Fig. 143 showing the section of a taper plug the difference in diameters for both tapers is $\frac{1}{2}$ in., but the taper of the hole is $\frac{1}{2}$ in. in 6 in. or 1 in. per ft. while the taper of the whole plug is $\frac{1}{2}$ in. per ft. The taper of

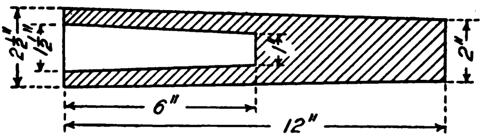


FIG. 143.

the hole is therefore twice as great as that of the whole plug.

In turning tapers in a lathe by setting over the tailstock we must take account of the *entire length* of the work between lathe centers although it may not all be tapered. For example, in Fig. 144, if we wish to turn the piece so that the taper of the 8-in. length shall be

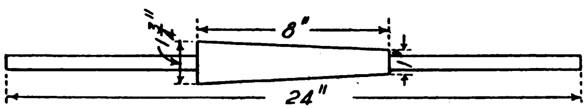


FIG. 144.

$1\frac{3}{4} - 1$ or $\frac{3}{4}$ in., since the whole piece is 24 in. long, the tailstock must be set over $\frac{1}{2}$ of $\frac{24}{8} \times \frac{3}{4}$ in. or $\frac{1}{2}$ of $\frac{9}{4}$ or $\frac{9}{8}$ or $1\frac{1}{8}$ in.

In other words, a taper of $\frac{3}{4}$ in. in 8 in. equals $\frac{1}{8}$ of $\frac{3}{4}$ or $\frac{3}{32}$ in. taper per in. which is $12 \times \frac{3}{32}$ or $\frac{9}{8}$ in. taper per ft. or $2 \times \frac{9}{8}$ or $\frac{18}{8}$ or $2\frac{1}{4}$ in. taper in 2 ft. Since the tailstock has to be set over a distance equal to *one-half* the total taper this distance equals ½ of 2¼ in. or 1⅛ in. The work will therefore appear set up as shown in Fig. 145. The cutting tool will move along the line *ab* cutting off ⅜ in. at the smaller end and running out at *a*.

In cutting tapers the point of the cutting tool must be set at the height of the lathe centers, otherwise as the diameter of the work becomes smaller there will be a change in the taper.

If the taper work is done by means of a taper attachment to a lathe the attachment must be set at an angle

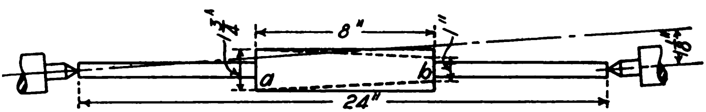


FIG. 145.

equal to one-half the total taper angle or at the angle of taper from the center line of the work. If in cutting tapers the compound tool rest is used, it must be set to the angle of taper from the center line.

PROBLEMS

8. A tapered piece 2 ft. long is 8 in. in diam. at one end and 12 in. in diam. at the larger end. What is the taper per in. and the taper per ft.?

9. A piece of stock 18 in. long is to be tapered the whole length to diameters at its ends of 1½ in. and 2¼ in. How far must the tailstock be set over to turn this taper?

10. What is the total taper in a pin 18 in. long whose taper is ⅜ in. per ft.?

11. For a taper of ½ in. per ft., what is the small diameter of a piece 4½ in. long tapered its whole length if its large diameter is 2¼ in.?

MISCELLANEOUS PROBLEMS

12. A white pine block is in the shape of the frustum of a square pyramid with an upper base 4 in. on a side, lower base, 8 in. on a side and altitude of 10 in. Find the weight of the block at 28 lb. per cu. ft. for white pine.

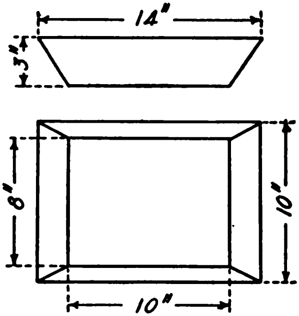


FIG. 146.

13. Find the contents in cu. in. of the drip pan shown in Fig. 146. How many gallons will the pan hold?

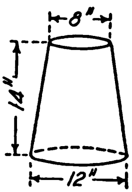


FIG. 147.

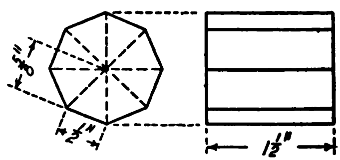


FIG. 148.

14. A water tank is 10 ft. in inside diam. at the top, and 12 ft. in inside diam. at the bottom. If its height is 8 ft. how many cu. ft. and how many gallons will it hold?

15. Find the weight of a steel ball 1 in. in diam.

16. Find the cost of painting a water tank 8 ft. in diam. at the top, 10 ft. in diam. at the bottom and 7 ft. high, if 1 gal. of paint costing $1.80 covers 300 sq. ft.

17. Find the weight of an oak block as per Fig. 147 if the weight per cu. ft. is 45 lb.

18. Find the weight of a cast-iron pin 8 in. long and with diameters of 1½ in. and 1 in. at the large and small ends respectively.

19. How many cubic inches in the piece of chisel steel represented in Fig. 148?

CHAPTER XVII

MENSURATION OF MISCELLANEOUS SOLIDS

70. The Wedge.—The *wedge* is a solid whose base is a rectangle, two of its opposite sides are parallelograms and two are parallel triangles, as in Fig. 149. $ABCD$ is the *base* and EF the *altitude*. The *entire area* of the *wedge* equals the sum of the areas of all its faces. The *volume* of the *wedge* equals the area of its base multiplied by one-half the altitude.

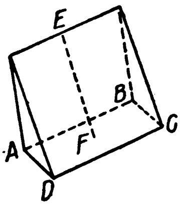


FIG. 149.

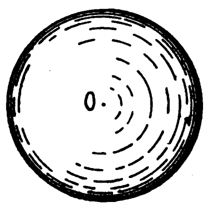


FIG. 150.

71. The Sphere.—The *sphere*, Fig. 150, is a solid bounded by a uniformly curved surface every point of which is equally distant from a point within called the *center*, as O. The *area* or the *surface* of a *sphere* equals the square of the diameter multiplied by Pi (π), that is 3.1416, or the *area* of a *sphere* $= \pi D^2$. The volume of a sphere equals $\frac{\pi}{6}$ multiplied by the *cube* of the diameter, or the volume $= \frac{\pi}{6}D^3$.

72. Area of a Hexagon and Volume of Solids with a Hexagonal Section.—In trade work it often becomes necessary to find the area of a hexagon, since this is the form of hexagonal nuts, bolts, and bar metal. Often times the problem is to find the length of a side of the hexagon when the area is given. To find the *area* of the hexagon shown in Fig. 151, we may divide it into six equilateral triangles 2 in. on a side. The area of each one of these triangles equals the product of the base of the triangle by half its altitude. If the triangle is 2 in. on each side the

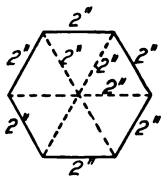


FIG. 151.

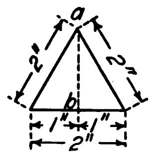


FIG. 152.

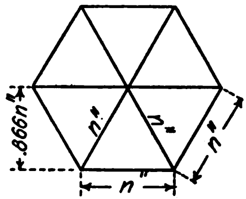


FIG. 153.

height *ab* of the triangle equals $\sqrt{(2)^2 - (1)^2}$ or $\sqrt{3}$ or 1.732 in. The area of the triangle then equals base $\times$ $\frac{\text{height}}{2}$ or

$$\overset{1}{\cancel{2}} \times \frac{1.732}{\underset{1}{\cancel{2}}} \text{ or } 1.732 \text{ sq. in.}$$

The area of the hexagon then equals 6×1.732 or 10.392 or 10.4 sq. in.

If we take a hexagon "*n*" in. on a side, as in Fig. 153, the area equals $6 \times$ area of one triangle or $6 \times \frac{n \times .866n}{2}$ (.866n is the height of each triangle) or $2.60n^2$ sq. in.

The area of a hexagon therefore equals $2.60 \times$ the square of one side.

Example.—How many square inches in the head of a hexagonal bolt $1\frac{1}{2}$ in. on a side.

$$\text{Area} = 2.60 \times 1\frac{1}{2} \times 1\frac{1}{2} \text{ or } 5.85 \text{ sq. in.}$$

If we have the area of a hexagon and wish to find the length of one side we divide the whole area by 6, giving the area of each of the equilateral triangles making up the hexagon. From the area of an equilateral triangle the length of a side is obtained from the rule

$$\text{Length of side} = \sqrt{\frac{\text{area of triangle}}{.433}}$$

or if the area of a hexagon = 2.60 × the square of a side, the length of a side equals $\sqrt{\dfrac{\text{area of hexagon}}{2.60}}$

Example.—The area of a hexagon equals 24 sq. in. What is the length of a side?

Ans. $\text{Length of side} = \sqrt{\dfrac{\text{area}}{2.60}} = \sqrt{\dfrac{24}{2.60}} = \sqrt{9.23} = 3.04 \text{ in.}$

PROBLEMS

1. How many square inches area in the head of a hexagonal bolt, the head being 0.5 in. on a side?

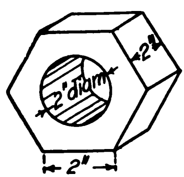


FIG. 154.

2. Find the volume in cu. in. and weight in lb. of 100 steel hexagonal nuts as per Fig. 154, the material weighing .28 lb. per cu. in.

3. Find the cost at 18 cts. per ft. of enclosing 1800 sq. ft. of land in the form of a regular hexagon.

4. Find the weight at .278 lb. per cu. in. of the hollow hexagonal bar shown in Fig. 155.

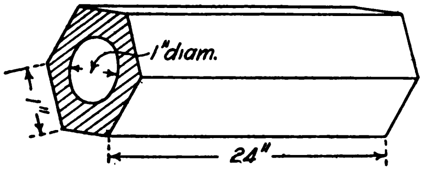


FIG. 155.

In the hexagon shown in Fig. 156, 1 in. on a side, each of the equilateral triangles has a true height or altitude of

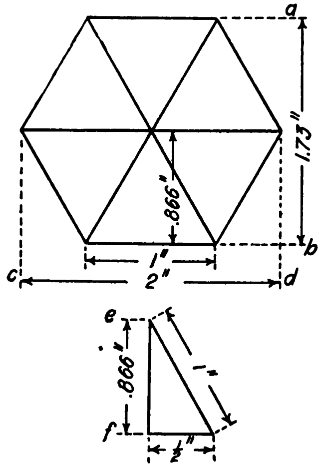


FIG. 156.

.866 in. The distance *ab* across flats equals 2 × .866 = 1.73 in. Also the distance *cd* across corners equals 2 × 1 or 2 in. Therefore the *ratio* of the distance across corners to the distance across flats $= \frac{2}{1.73} = 1.15$. This is therefore the rule for finding the distance across corners of a hexagonal nut or bolt head when we have the distance across flats. *The distance across corners of a hexagonal nut or bolt head equals the distance across flats* $\times$ 1.15.

According to the figure shown,

$$ef = \sqrt{1 \times 1 - \tfrac{1}{2} \times \tfrac{1}{2}} = \sqrt{1 - \tfrac{1}{4}}$$
$$= \sqrt{\tfrac{3}{4}} = \sqrt{.75} = .866$$

The distance across flats on a standard hexagonal bolt or nut equals $1\frac{1}{2}$ times the diameter of the bolt plus $\frac{1}{8}$ in., or if D is the bolt diameter, F the distance across flats and C the distance across corners, we have the rules

$$F = 1\tfrac{1}{2}D + \tfrac{1}{8} \text{ in.}$$
$$C = F \times 1.15$$

PROBLEMS

5. Find the distance across flats and across corners of the head of a 1-in. diam. bolt.

6. Find the distance across flats and corners of a nut to work on a $2\frac{1}{2}$-in. diam. bolt.

7. Find the weight of the hollow steel bar shown in Fig. 157 if the weight per cu. in. is .28 lb.

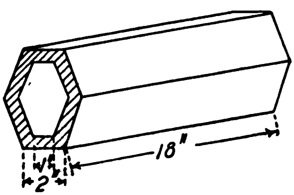


Fig. 157.

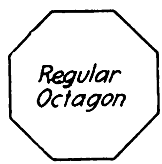


Fig. 158.

73. The Octagon.—Figure 158 shows an "octagon" or eight-sided figure. Chisel steel comes in this form, and also nuts for pipe unions are made in this form, so that they can be turned up a little at a time when they are being put on at a place difficult to reach. The area of the octagon is found from the rule

Area = the square of a side × 4.828

Each of its angles equals 135° or 90° + 45°.

Example.—Find the weight of a piece of octagonal chisel steel $\frac{3}{8}$ in. on a side and 18 in. long, if the steel weighs .283 lb. per cu. in.

Solution.—Wt. = Area of cross section × length × weight per cu. in.

$= \frac{3}{8} \times \frac{3}{8} \times 4.828 \times 18 \times .283 = 3.47$ lb.

8. Find the weight of a piece of octagonal chisel steel .5 in. on a side and 2 ft. long, if the steel weighs .283 lb. per cu. in.

74. STANDARD SECTIONS USED IN CONSTRUCTION

In work of construction the following shaped sections are used in beams, supports and braces of various kinds. Sometimes "built-up" sections are used by combining two or more of the sections shown in Figs. 159 and 160.

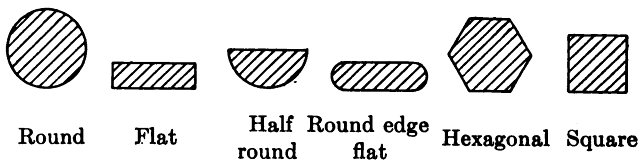


FIG. 159.

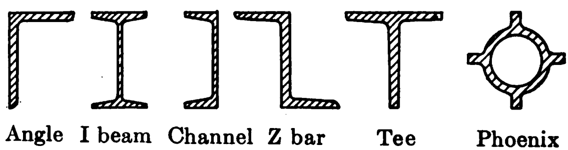


FIG. 160.

CHAPTER XVIII

MISCELLANEOUS RULES FOR POLYGONS

75. The Equilateral Triangle. (Equal Sided).—Suppose the length of the side of the triangle as shown in Fig. 161 is "a" in. Drawing in the altitude divides the triangle into two equal right triangles whose bases are $\frac{a}{2}$ in.

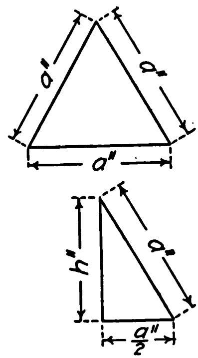


FIG. 161.

The height "h" of each of these triangles (from the rule for the sides of a right triangle) is found as follows:

$$h = \sqrt{a^2 - \left(\frac{a}{2}\right)^2} \text{ or } \sqrt{a^2 - \frac{a^2}{4}}$$

this equals $\sqrt{\tfrac{3}{4}a^2}$ or $.866a$.

That is, the *height* or *altitude* of *any* equilateral triangle equals .866 × the length of a side.

The area of any triangle equals $\frac{\text{base} \times \text{altitude}}{2}$, therefore the area of this equilateral triangle (substituting for the height its equal $.866a$ and for the base its value a in.) is

$$\text{Area} = \frac{a \times .866a}{2} \text{ or } .433a^2$$

That is, the area of any *equilateral* triangle equals .433 × the *square* of a side.

Example.—Find the area of an equilateral triangle 8 in. on a side.

Solution.—Area = .433 × (side)^2
or

$$\text{Area} = .433 \times 8 \times 8 \text{ or } 27.7 \text{ sq. in.}$$

76. Area of a Triangle from the Lengths of the Three Sides.—If only the three sides of a triangle are known, its area may be found as follows:

If the sides of the triangle are a, b, and c, as shown in Fig. 162, let "s" equal one-half the sum of the sides or

$$s = \frac{a + b + c}{2}$$

The rule then is

$$\text{Area} = \sqrt{s(s - a)(s - b)(s - c)}$$

Example.—Find the area of a triangular steel plate whose sides are 14, 10 and 8 in. long.

Solution:

$$s = \frac{14 + 10 + 8}{2} \text{ or } 16$$

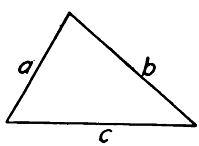


Fig. 162.

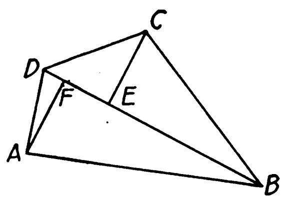


Fig. 163.

Then the area $= \sqrt{16(16 - 14)\ (16 - 10)\ (16 - 8)}$ or $\sqrt{16 \times 2 \times 6 \times 8}$ which equals $\sqrt{1536}$ or 39.19 sq. in.

77. Area of an Irregular Polygon.—In the case of an irregular polygon as shown in Fig. 163, its area may be found by drawing in the diagonal BD and the two perpendiculars to it CE and AF from the vertices C and A.

The area of the polygon then equals the sum of the areas of triangles CBD and ABD. Measuring BD which is the common base of the triangles and the heights CE and AF, the area of the figure equals

$$BD \times \frac{CE}{2} + BD \times \frac{AF}{2} \text{ or } BD \times \left(\frac{CE + AF}{2}\right)$$

In a similar way the area of any irregular figure bounded by straight lines may be found by dividing it into a number of simple figures such as triangles and rectangles and taking the sum of these simple areas as the area of the whole figure.

78. Area of any Regular (or Equal Sided) Polygon.— The area of any *regular* polygon can be expressed as a *constant* or *fixed* number times the *square* of a *side* of the polygon.

In the case of a regular or equilateral triangle the area has been shown to equal .433 × (side of triangle).^{2} The *constant* in this case is .433. For a *regular hexagon* the constant is 2.60.

From the constants in the following table we can calculate the area of any regular polygon by squaring its side and multiplying by the number given in the table for the particular polygon in question.

Constants Used in Finding the Areas of Regular Polygons

Number of sides	Name of polygon	Constant
3	Triangle	.433
4	Square	1.000
5	Pentagon	1.720
6	Hexagon	2.598
7	Heptagon	3.634
8	Octagon	4.828
9	Nonagon	6.182
10	Decagon	7.694
11	Undecagon	9.366
12	Dodecagon	11.196

Example.—Find the area of cross section of an octagonal steel bar 3/8 in. on a side.

Solution.—Area = 4.828 × (side)^2 or 4.828 × (.375)^2 which equals 0.679 sq. in. *Ans.*

MISCELLANEOUS PROBLEMS FOR PRACTICE

1. A steel plate in the form of an equilateral triangle is 2 ft. 8 in. on a side. Find its area in sq. in. If the plate is 3/8 in. thick, find its weight. (Steel weighs 0.28 lb. per cu. in.)

2. Find the area of a piece of triangular sheet steel whose sides measure 8.4 in., 6.8 in. and 10.2 in.

3. In measuring a triangular piece of land the length of its sides are found to be 44 ft., 86 ft., and 98 ft. What is the area of the land in sq. ft.?

4. Draw an irregular four-sided polygon whose sides taken in order are 3/4 in., 1 1/4 in., 1 3/4 in., and 5/8 in. Draw in a diagonal and perpendiculars from two of the angles to the diagonal, dividing the figure into four triangles. Measure the lengths of the bases and altitudes of these triangles and from these figures calculate the total area of the polygon. Have the first two sides form a right angle.

5. Using the table of constants given, find the area of a regular hexagon 1 3/8 in. on a side.

6. Find the weight of 6 ft. of an octagonal steel bar 5 in. on a side. (Use table of constants for finding cross section of bar.) (Steel weighs .283 lb. per cu. in.)

CHAPTER XIX

MISCELLANEOUS RULES FOR LENGTH, AREA AND VOLUME

79. Calculation of Area of an Irregular Figure.—In Fig. 164 is shown an indicator card such as is obtained on a steam indicator in finding the horsepower of a steam or gas engine. Since this figure cannot be divided into simple areas such as triangles and rectangles, an approximate method for finding its area is as follows:

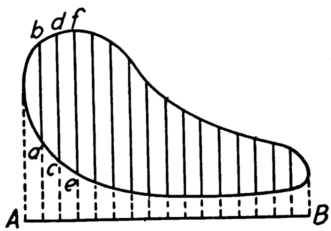


FIG. 164.

Draw a line AB projected from the ends of the given area. Divide this line into any number of equal parts, in this case 16, and extend lines across the given area.

Then find the sum of the lengths ab, cd, ef, etc., and divide by the number of lines, thus obtaining the *average height* of the figure. This *average height* multiplied by the *length* of the figure gives its *area* very closely. The more lines used in this method the more correct will be the answer.

Another method for finding the area of an irregular figure is to cut it out of cardboard and compare its weight with the weight of a square inch of the same thickness of cardboard.

Still another method is to use the "Planimeter" an instrument from which we may obtain the area of a figure directly. This instrument is described in the second volume of this work, that is the volume on "Practical Mechanics and Allied Subjects."

80. Area of the Sector of a Circle.—The *sector* of a circle is the portion of its area included between two of

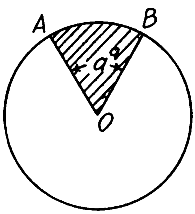


FIG. 165.

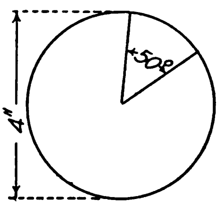


FIG. 166.

its radii and the portion of its circumference intercepted by the angle which the radii form. In Fig. 165 the area AOB is a *sector* of the circle whose center is O and radii AO and BO. If "a" is the angle of the sector in degrees, the area of the sector equals $\frac{a}{360} \times$ area of entire circle.

Example.—Find the area of a sector whose angle is 50° in a circle 4 in. in diam., as shown in Fig. 166.

Solution.—Area $= \frac{50}{360} \times 4 \times 4 \times .785$ or 1.745 sq. in.

81. Area of the Segment of a Circle.—The *segment* of a circle is the portion of its area included between a *chord* of the circle and the portion of the circumference subtended by the *chord*. In Fig. 167 the *chord* AB subtends the arc ACB. The segment is area ABC shown cross sectioned. The area of the segment equals the difference between the areas of sector $AOBC$ and triangle AOB.

Example.—If Fig. 167 represents the cross section of a boiler 78 in. in diam., and AB represents the "water line," find the area of the steam segment ABC.

Solution.—Area of segment = area of sector − area of triangle or $\frac{90}{360} \times 78 \times 78 \times .785 - \frac{39}{2} \times 39$ which equals 1193.9 − 760.5, or 433.4 sq. in.

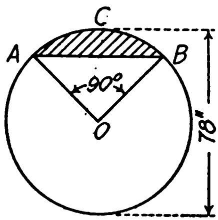


FIG. 167.

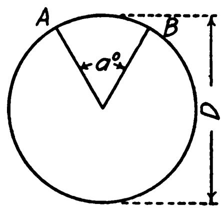


FIG. 168.

NOTE.—The base and height of the triangle each equal the *radius* of the circle or 39 in.

82. Length of the Arc of a Circle.—The length of the arc of a circle subtended by a given central angle as angle "*a*," in Fig. 168, equals the whole circumference times the ratio of the given central angle to 360° or written in the form of an equation, arc $AB = \pi D \times \frac{a°}{360°}$ in which D is the diameter of the circle and π (Pi) is the constant 3.1416.

83. Area of a Surface Forming a Ring.—The area of the *ring*, which is the area between the circumferences of the two circles shown in Fig. 169, or the area shown cross sectioned, equals the *difference* between the areas of the large and small circles. If the diameters of these circles are denoted by D and d, the area of the ring $= .785D^2 - .785d^2$ or $.785\ (D^2 - d^2)$. Since $.785 = \frac{\pi}{4}$, this rule may also be written. Area of ring $= \frac{\pi}{4}\ (D^2 - d^2)$.

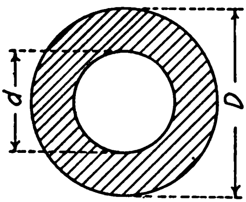


Fig. 169.

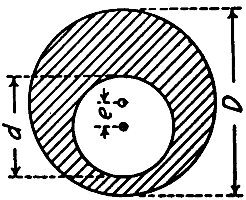


Fig. 170.

This rule holds true whether the circles are *concentric*, that is, having the same center as shown in Fig. 169, or *eccentric*, that is, having different centers as shown in Fig. 170. In this figure the distance between centers or the *eccentricity* is the distance "*e*."

84. The Ellipse.—The ellipse is a plane figure bounded by a curved line such that the sum of the distances of any point in the bounding line from two fixed points called *foci* (singular focus) is always the same, or as we say a *constant*.

Figure 171 shows an ellipse, for which the sum of the distances of point A from the foci F and F', that is, $AF + AF'$, is equal to the sum of the distances of *any other point in the bounding line* as point B from F and F'.

That is, $AF + AF' = BF + BF'$. Point O is the *center* of the ellipse, CD is the *Major axis,* and GH the *Minor axis.*

OG and OD are the *semi-axes* of the ellipse. If the major and minor axes of an ellipse are denoted by "a"

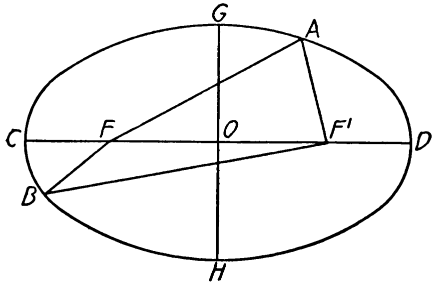


Fig. 171.

and "b" respectively as shown in Fig. 172 the *circumference* of the ellipse is found as follows:

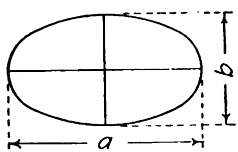


Fig. 172.

$$\text{Circumference} = \pi\sqrt{\frac{a^2 + b^2}{2}} \text{ (approximately)}$$

in which

$$\pi = 3.1416 \text{ and is a constant.}$$

The *area* of the ellipse in terms of its axes is found as follows:

$$\text{Area} = \frac{\pi}{4}\, ab, \text{ or } .785ab.$$

Example.—A plot of ground is laid out in elliptical shape having diameters of 12 ft. and 8 ft. Find the distance around the plot and its area in sq. ft.

Solution.—Circumference $= \pi\sqrt{\frac{a^2 + b^2}{2}}$ or $3.1416 \times \sqrt{\frac{(12)^2 + (8)^2}{2}}$ which equals $3.1416 \times \sqrt{104}$ or 32.0 ft., approximately.

The area $= .785ab$ or $.785 \times 12 \times 8$ which equals 75.4 sq. ft.

85. Use of the Square.—The square may be used to find the diameter of a circle having the same area as two

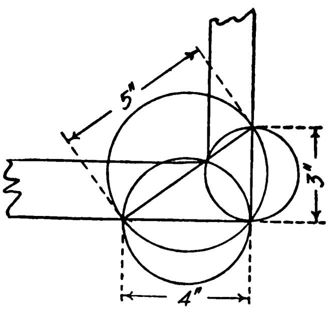


FIG. 173.

other circles added together. If the two circles are 4 in. and 3 in. in diam., we lay off on one arm of the square 4 in., and on the other 3 in. as shown in Fig. 173. The line connecting these points equals the diameter of the equivalent circle.

Proof.—The connecting line is 5 in. long, since it is the hypotenuse of a right triangle of which 4 in. and 3 in. are the other two sides. Therefore $(4)^2 + (3)^2 = 25$ or 5^2, and the area of a 5-in. circle equals the sum of the areas of a 4-in. and a 3-in. circle as shown below.

$$5 \times 5 \times .785 = 4 \times 4 \times .785 + 3 \times 3 \times .785$$

that is

$$19.62 = 12.56 + 7.06$$

or

$$19.62 = 19.62.$$

This method can be applied to any regular plane figures, such as hexagons, octagons and so forth.

86. Rules for Calculating Lengths of Belts.—Since pulleys are circular in shape the rules for calculating

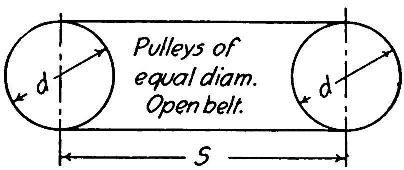


FIG. 174.

lengths of belting required to connect two pulleys, involve the rules for a circle.

Case I.—When pulleys are of *equal* diameter.

Open Belt.—From Fig. 174 it can be seen that the length of belt required equals *twice the distance between pulley centers plus the entire circumference of one pulley.* If "l" is the length of belt $l = \pi d + 2s$.

Case II.—When pulleys are of *unequal* diameter.

Open Belt.—If the pulleys as shown in Fig. 175 are of unequal size, but the difference in diameter is not great the length of belt is given approximately by the equation,

$$l = \pi\left(\frac{D + d}{2}\right) + 2s$$

A more general equation giving closer results is as follows:

$$l = 2\sqrt{\frac{(D-d)^2}{2} + s^2} + \pi\left(\frac{D+d}{2}\right)$$

Case III.—When pulleys are of *equal* diameter.

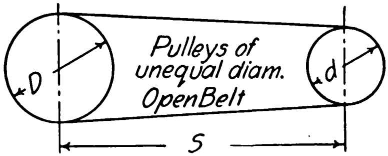


FIG. 175.

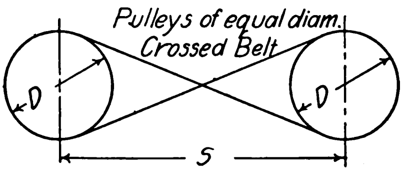


FIG. 176.

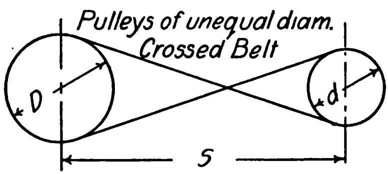


FIG. 177.

Crossed Belt.—In this case, Fig. 176, the length "l" is given by the rule

$$l = 2\sqrt{D^2 + s^2} + \pi D$$

Case IV.—When pulleys are of *unequal* diameter.

Crossed Belt.—For this case, Fig. 177, the length

$$l = 2\sqrt{\left(\frac{D+d}{2}\right)^2 + s^2} + \pi\left(\frac{D+d}{2}\right)$$

87. Table of Values for Spacing Circles.—It is often necessary to divide a circle into a given number of equal parts, such, for example, as when spacing the centers for stud holes on a cylinder head. In the case of *six* equally spaced centers on a given circumference, the chord used, or the distance taken on the dividers equals the *radius* or *half the diameter* of the circle, that is, in Fig. 178 distances AB and OC are equal. To space *seven* holes, however, the chord is not found as easily. The following table gives the length of chord for setting of the dividers for spacing a circle 1 in. in diam. into any number of

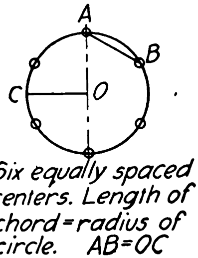


Fig. 178.

equal parts from 3 to 82 inclusive. The numbers for the chords given in the table therefore represent decimal parts of an inch.

For circles of diameter larger or smaller than 1 in., multiply the value in the table by the diameter of the given circle.

For example, to space a 20-in. diam. circle into 14 equal parts, we find in the table opposite 14 the decimal .2225 which is the correct setting of the dividers for a 1-in. circle. For the 20-in. circle we take 20 times .2225 in. or 4.450 in. as the correct setting of the dividers.

LENGTH OF CHORDS FOR SPACING A 1-IN. DIAM. CIRCLE

No. of spaces	Length of chord	No. of spaces	Length of chord	No. of spaces	Length of chord	No of spaces	Length of chord
3	.8660	23	.1362	43	.0730	63	.0498
4	.7071	24	.1305	44	.0713	64	.0491
5	.5878	25	.1253	45	.0698	65	.0483
6	.5000	26	.1205	46	.0682	66	.0476
7	.4339	27	.1161	47	.0668	67	.0469
8	.3827	28	.1120	48	.0654	68	.0462
9	.3420	29	.1081	49	.0641	69	.0455
10	.3090	30	.1045	50	.0628	70	.0449
11	.2817	31	.1012	51	.0616	71	.0442
12	.2588	32	.0980	52	.0604	72	.0436
13	.2393	33	.0951	53	.0592	73	.0430
14	.2225	34	.0923	54	.0581	74	.0424
15	.2079	35	.0896	55	.0571	75	.0419
16	.1951	36	.0872	56	.0561	76	.0413
17	.1837	37	.0848	57	.0551	77	.0408
18	.1736	38	.0826	58	.0541	78	.0403
19	.1646	39	.0805	59	.0532	79	.0398
20	.1564	40	.0785	60	.0523	80	.0393
21	.1490	41	.0765	61	.0515	81	.0388
22	.1423	42	.0747	62	.0507	82	.0383

The figures in the above table apply in general for a diameter of 1 or *unity*, that is 1 *inch*, 1 *foot*, 1 *yard* and so forth, provided we remember that the length of the *chord* is in the *same unit* as the *diameter*. For example, referring to the table to lay off 34 equal spaces on a 1-*foot* circle, the dividers must be set to .0923 *foot.*

88. Volume and Area of Solid Rings.—Figure 179 shows a solid ring with a *circular* cross section. The *length* of the ring is the length of the dotted line or circumference drawn through the center of the cross section of the ring. The solid ring may be considered as generated by the rotation of the circular section whose center is O' and diameter d about the horizontal axis AB. The axis AB is at right angles to the plane of the ring.

The entire *area* or *surface* of the ring equals the *circumference* of the cross section of the ring times the length of the dotted circle shown of diameter "D"—that is

Entire *area* of ring $= \pi d \times \pi D$ or $\pi^2 dD$

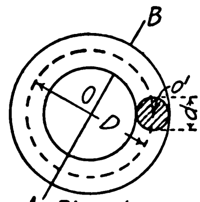


FIG. 179.

The *volume* of the ring equals the *area of cross section* of the ring times the length of the dotted circle shown of diameter "D," that is,

$$\text{Volume of ring} = \frac{\pi d^2}{4} \times \pi D \text{ or } \frac{\pi^2 d^2 D}{4}$$

In general for a circular ring of any shaped cross section its *entire area* equals the product of the *perimeter* of its section times the circumference of the circle drawn through the *center of gravity* of its cross section. The *center* of gravity of the section is the point in the section on which it would balance if made of cardboard of the given shape.

The *volume* of a ring of any shaped cross section equals the product of the *area of* its cross section times the circumference of the circle drawn through the *center* of *gravity* of its cross section.

1. Find the area of the indicator card shown in Fig. 180 by using the method explained in article 77, that is, by measurements taken on the figure. (Read distances to the nearest 32nd of an inch.)

2. Figure 181 shows the cross section of a 60-in. diam. boiler. If the water line AB is 44 in., find the area of the steam segment ACB

and the area of the water segment *AEB*. (Note that *OA* and *OB* are each radii of the circle.)

3. Find the area of a segment in a 4-in. diam. circle, if the chord of the segment is formed by connecting the ends of radii making a 60° central angle.

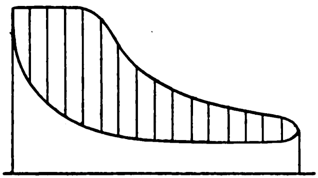


Fig. 180.

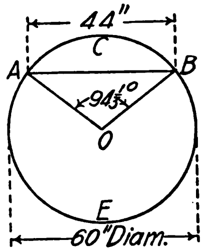


Fig. 181.

4. What length of arc is intercepted by a 29° central angle in a circle whose diameter is 2.8 in.?

5. If the minute hand on a tower clock is 6½ ft. long, through what distance does its end move from 10 : 10 A. M. to 10 : 24 A. M.?

6. Find the area of cross section of a hollow cast-iron column of

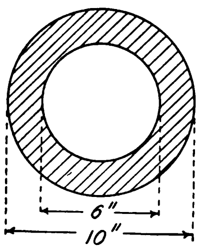


Fig. 182.

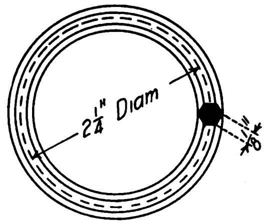


Fig. 183.

dimensions shown in Fig. 182. If the column is 14 ft. long, find its weight if cast iron weighs .26 lb. per cu. in.

7. Find the circumference and area of an ellipse whose major and minor axes are respectively 12 in. and 7 in.

8. Find the length of belt required to connect two 18-in. pulleys whose centers are 8 ft. apart. Open belt.

9. Find the length of belt required for the above pulleys if the belt is crossed.

10. What length of belt is required to connect a 24-in. pulley and a 12-in. pulley with an open belt if the pulley centers are 8 ft. 6 in. apart?

11. Find the length of belt required for the above pulleys if the belt is crossed.

12. What distance would you take on dividers for spacing 22 centers on a 14-in. diam. circle?

13. Find the volume and area of the hexagonal ring shown in Fig. 183. Find its weight if made of steel weighing .284 lb. per cu. in.

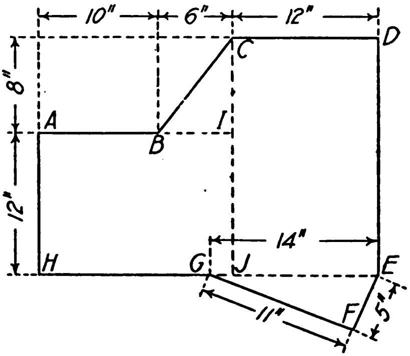


FIG. 184.

14. Find the area of a piece of sheet tin of the dimensions shown in Fig. 184, by finding first the areas of rectangles *AIJH* and *CDEJ* and adding to these the areas of triangles *BCI* and *EFG*.

CHAPTER XX

THE METRIC SYSTEM OF MEASUREMENT

89. Definition.—In addition to the English System of measurement having as standards of length the *yard*, *foot*, *inch*, and so forth, and for units of weight the *ton*, *pound*, and *ounce*, there is a system called the *Metric System* having as a standard of length the *meter* which is a little longer than the *yard*. It contains 39.37 in. This same system has as a unit of weight the *gram*, which equals $\frac{1}{453.6}$ part of a pound.

The meter is divided into one hundred equal parts called centimeters, or hundredths of a meter. Ten of these centimeters make $^{10}/_{100}$ or $^{1}/_{10}$ meter and are called a *Decimeter*. For fine measurements of weight the centigram or $^{1}/_{100}$ gram and the milligram or $^{1}/_{1000}$ gram are used. For larger measurements of weight the kilogram or 1000 gram weight is used.

The measurement of areas and volumes are made in terms of units of volume derived from the units of length described above.

The metric system is that used in science, that is physics and chemistry, and in some countries is the system used for common as well as scientific measurements.

90. The Metric Units.—The metric units are as follows:

Unit of length	meter
Unit of area	are
Unit of volume	liter
Unit of weight	gram

The liter is the volume of a cube whose linear dimensions are each 10 centimeters or 1 decimeter. The volume of the liter is therefore 10^3 or 1000 cubic centimeters.

One cubic centimeter of pure water weighs 1 gram, and one liter weighs 1000 grams or 1 kilogram.

The metric system is especially useful since it is based on the decimal system or system of tens.

The following terms are used to designate the various relations of the units.

Mikro denotes $\frac{1}{1,000,000}$ or .000001.

Milli denotes $\frac{1}{1000}$ or .001.

Centi denotes $\frac{1}{100}$ or .01.

Deci denotes $\frac{1}{10}$ or .1.

Deka denotes 10

Hecto denotes 100.

Kilo denotes 1000.

Myria denotes 10,000.

91. Metric Tables.—Following are the Metric Tables of Length, Area, Volume, and Weight.

LINEAR MEASURE

10 millimeters(mm.)	=1 centimeter(cm.)	=.1 decimeter =.01 m.
10 centimeters	=1 decimeter	=0.1 meter
10 decimeters	=1 meter	
10 meters	=1 dekameter (Dm.)	=10 meters
10 dekameters	=1 hectometer (Hm.)	=100 meters
10 hectometers	=1 kilometer (Km.)	=1000 meters
10 kilometers	=1 myriameter (Mm.)	=10,000 meters

SURFACE MEASURE

100 sq. millimeters ($mm.^2$)	= 1 sq. centimeter ($cm.^2$)
100 sq. centimeters	= 1 sq. decimeter ($dm.^2$)
100 sq. decimeters	= 1 sq. meter ($m.^2$) or 1 centare (ca.)
100 sq. meters	= 1 sq. dekameter ($Dm.^2$) = 1 arc (a.)
100 sq. dekameters	= 1 sq. hectometer ($Hm.^2$) = 1 hektare (Ha.)
100 sq. hectometers	= 1 sq. kilometer ($Km.^2$)

VOLUME MEASURE

1000 cu. millimeters ($mm.^3$)	= 1 cu. centimeter ($cm.^3$) or (c.c.)
1000 cu. centimeters	= 1 cu. decimeter ($dm.^3$) or 1 liter (l.)
1000 cu. decimeters	= 1 cu. meter ($m.^3$) or 1 kiloliter(kl.)

WEIGHT MEASURE

10 milligrams	= 1 centigram (cg.)
10 centigrams	= 1 decigram (dg.)
10 decigrams	= 1 gram (g.)
10 grams	= 1 dekagram (Dg.)
10 dekagrams	= 1 hectogram (Hg.)
10 hectograms	= 1 kilogram or kilo (Kg.)
10 kilograms	= 1 myriagram (Mg.)
10 myriagrams	= 1 quintal (Q.)
10 quintals	= 1 millier or metric ton (T.)

92. Comparison of Metric and English Systems.—Figure 185 shows two scales, one graduated in inches and the other in centimeters. This figure should be convenient for reference in studying the relation between the Metric and English systems. From the figure it is easily seen that 1 inch equals a little over 2.5 cm. To be more exact 1 in. = 2.540 cm.

The following table of Equivalents is given for convenience in referring one system to the other.

LINEAR MEASURE

1 inch	= 2.540 cm.	= .02540 m.		
1 foot	= 30.48 cm.	= .3048 m.		
1 yard	= 91.44 cm.	= .9144 m.		
1 rod	= 502.9 cm.	= 5.029 m.		
1 mile	= 160935cm.	= 1609.35 m.	1.60935 km.	
1 meter	= 39.37 in.	= 3.28 ft.	= 1.09 yd.	

SQUARE MEASURE

1 sq. inch	= 6.452 cm.^{2}	
1 sq. foot	= 0.09290 m.^{2}	
1 sq. yard	= 0.83613 m.^{2}	
1 sq. meter	= 10.76 sq. ft.	= 1.196 sq. yd.

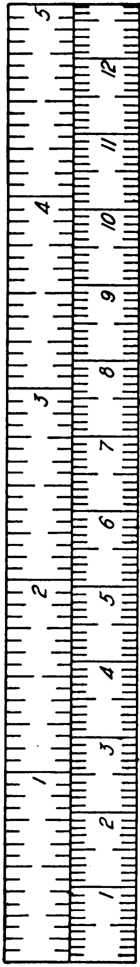
VOLUME MEASURE

1 cu. in.	= 16.39 c.c.
1 cu. ft.	= 28.32 dm.^{3} or liters.
1 qt. (dry)	= 1.101 dm.^{3} or liters.
1 qt. (liquid)	= 0.9464 dm.^{3} or liters.
1 cm.^{3}	= 0.0610 cu. in.
1 liter	= 61.02 cu. in.
1 liter	= 0.9081 qt. (dry)
1 liter	= 1.057 qt. (liquid).

WEIGHT MEASURE

1 grain	= 0.06480 grams.
1 ounce (avoirdupois)	= 28.35 grams.
1 pound (avoirdupois)	= 453.6 grams = .4536 kg.
1 ton (short)	= 907.2 kg.
1 gram	= 15.43 grains.
1 kilogram	= 2.205 pounds (avoirdupois).
1 metric ton	= 1000 kg. = 2204.6 pounds (avoirdupois).

It should be noticed that since the metric units are based on the decimal relations of numbers, that is on numbers which are multiples of 10, it is very easy to change from one unit to another. For example, 25.4 meters = 2540 centimeters. That is, since the centimeter is only 1/100 meter there are 100 times more centimeters in a given length than there are meters in the same length, and centimeters equal meters times 100. To multiply a number by 100 merely move the decimal point *two* places *to the right*, as was done above. For another example, how many millimeters in 2.493 meters. Here we multiply by 1000, that is move the decimal point *three* places *to the right* and we get 2493 millimeters for the answer. To change from a *smaller* to a *larger* unit in the metric system we divide by some multiple of 10, that is, move the decimal point *to the left*. For example: How many meters are there in 429,683 millimeters? Here we divide by 1000, that is move the decimal point *three* places *to the left* and get 429.683 meters, which is the answer.



Scales showing the Relation between Centimeters and Inches

FIG. 185.

In working with square measure we should remember and *square* the ratio of the equivalent units in linear measure, for example, 1 meter = 100 centimeters. While 1 square meter = 100 × 100 or 10,000 square centimeters.

Also in cubic measure we should remember to *cube* the ratio of the equivalent units in linear measure, for example:

1 meter = 1000 millimeters, while 1 cubic meter = 1000 × 1000 × 1000 or 1,000,000,000 cubic millimeters.

In changing from the English to the Metric system or vice versa, it is merely a matter of multiplication or division as the case may be, for example:

To change 20 ft. to meters,
since 1 foot = .3048 meters.

20 feet = 20 × .3048 or 6.096 meters.

To change 2493 kilograms to tons
since 1 ton = 907.2 kg.

$$2493 \text{ kilograms} = \frac{2493}{907.2} \text{ or } 2.75 \text{ tons.}$$

PROBLEMS

1. Write the following in cm.: 2.92 m., 93.2 dm., .700 km., 493 mm.

2. What is the size in in. of a 40-cm. gun?

3. Write the following in sq. mm.: 82 dm.^{2}, 3.26 m.^{2}, .426 km.^{2}, 27 cm.^{2}.

4. How many square meters of land in a lot containing 22 sq. rods? How many square decimeters in the same lot?

5. Change the following to cubic meters: 143 l., 9438 dm.^{3}, 428 cm.^{3}, 9.4 kl.

6. Change 43 l. to dry quarts and also to liquid quarts. Find the number of kl. in 273 dry quarts and in 698.2 liquid quarts.

7. A rectangular tank is 2.5 m. long, 1.4 m. wide and .92 dm. deep. Find its capacity in liters. Find the weight of water it will hold when full if 1 cu. cm. of water weighs 1 gram.

8. Find the difference in cm. between the lengths of two steel rods one of which is 2.79 m. long and the second 18.4 in. in length.

9. A speed of 200 ft. per sec. is how many kilometers per sec.?

10. When an object falls freely it increases in speed each second 32.2 ft. per sec. Express this in cm. per sec. each second.

11. An express train is traveling at the rate of 60 miles per hour. Find its speed in kilometers per minute.

12. The thickness of a steel plate is $\frac{3}{8}$ in. Find this thickness in cm. and in dm. If the plate has an area of 420 dm.^{2}, find its volume in cu. in. and its weight in lb. if 1 cu. in. of steel weighs 0.283 lb.

13. Find the number of liters in a tank containing 24 barrels. (One barrel contains 31.5 gal.)

14. A piece of steel bar is laid off to a length of 432 cm. Find this length in feet and inches.

15. A block of granite weighs 2.4 tons. Find its weight in kilograms.

16. Find the weight in grams of the air in a room 10 ft. × 12 ft. and 11 ft. high, if the air is .00128 times as heavy as water. (Water weighs 62.5 lb. per cu. ft.)

TABLES

TABLE I

DECIMAL EQUIVALENTS, SQUARES, SQUARE ROOTS, CUBES AND CUBE ROOTS OF FRACTIONS; CIRCUMFERENCES AND AREAS OF CIRCLES FROM 1/64 TO 1 INCH

Fraction	Dec. Equiv.	Square	Sq. Root	Cube	Cube Root	Circum. Circle	Area Circle
1/64	.015625	.000244	.1250	.0000003815	.2500	.04909	.000192
1/32	.03125	.0009765	.1768	.00003052	.3150	.09818	.000767
3/64	.046875	.002197	.2165	.000103	.3606	.1473	.001726
1/16	.0625	.003906	.2500	.0002442	.3968	.1963	.003068
5/64	.078125	.006104	.2795	.0004768	.4275	.2455	.004794
3/32	.09375	.008789	.3062	.0008240	.4543	.2945	.006903
7/64	.109375	.01196	.3307	.001308	.4782	.3436	.009396
1/8	.1250	.01563	.3535	.001953	.5000	.3927	.01228
9/64	.140625	.01978	.3750	.002781	.5200	.4438	.01553
5/32	.15625	.02441	.3953	.003815	.5386	.4909	.01916
11/64	.171875	.02954	.4161	.005078	.5560	.5400	.02321
3/16	.1875	.03516	.4330	.006592	.5724	.5890	.02761
13/64	.203125	.04126	.4507	.008381	.5878	.6381	.03241
7/32	.21875	.04786	.4677	.01047	.6025	.6872	.03758
15/64	.234375	.05493	.4841	.01287	.6166	.7363	.04314
1/4	.2500	.0625	.5000	.01562	.6300	.7854	.04909
17/64	.265625	.07056	.5154	.01874	.6428	.8345	.05541
9/32	.28125	.07910	.5303	.02225	.6552	.8836	.06213
19/64	.296875	.08813	.5449	.02616	.6671	.9327	.06922
5/16	.3125	.09766	.5590	.03052	.6786	.9817	.07670
21/64	.328125	.1077	.5728	.03533	.6897	1.031	.08456
11/32	.34375	.1182	.5863	.04062	.7005	1.080	.09281
23/64	.359375	.12913	.5995	.04641	.7110	1.129	.1014
3/8	.3750	.1406	.6124	.05273	.7211	1.178	.1104
25/64	.390625	.1526	.6250	.05960	.7310	1.227	.1226
13/32	.40625	.1650	.6374	.06705	.7406	1.276	.1296
27/64	.421875	.17800	.6495	.07508	.7500	1.325	.1398
7/16	.4375	.1914	.6614	.08374	.7592	1.374	.1503
29/64	.453125	.2053	.6732	.09304	.7681	1.424	.1613
15/32	.46875	.2197	.6847	.1030	.7768	1.473	.1726
31/64	.484375	.2346	.6960	.1136	.7853	1.522	.1843
1/2	.5000	.2500	.7071	.1250	.7937	1.571	.1963

TABLE I—*Concluded*

DECIMAL EQUIVALENTS, SQUARES, SQUARE ROOTS, CUBES, CUBE ROOTS OF FRACTIONS; CIRCUMFERENCES AND AREAS OF CIRCLES FROM 1/64 TO 1 INCH

Fraction	Dec. Equiv.	Square	Sq. Root	Cube	Cube Root	Circum. Circle	Area Circle
33/64	.515625	.2659	.7181	.1371	.8019	1.620	.2088
17/32	.53125	.2822	.7289	.1499	.8099	1.669	.2217
35/64	.546875	.2991	.7395	.1636	.8178	1.718	.2349
9/16	.5625	.3164	.7500	.1780	.8255	1.767	.2485
37/64	.578125	.3342	.7603	.1932	.8331	1.816	.2625
19/32	.59375	.3525	.7706	.2093	.8405	1.865	.2769
39/64	.609375	.3713	.7806	.2263	.8478	1.914	.2916
5/8	.6250	.3906	.7906	.2441	.8550	1.963	.3068
41/64	.640625	.4104	.8004	.2629	.8621	2.013	.3223
21/32	.65625	.4307	.8101	.2826	.8690	2.062	.3382
43/64	.671875	.4514	.8197	.3033	.8758	2.111	.3545
11/16	.6875	.4727	.8292	.3250	.8826	2.160	.3712
45/64	.703125	.4944	.8385	.3476	.8892	2.209	.3883
23/32	.71875	.5166	.8478	.3713	.8958	2.258	.4057
47/64	.734375	.5393	.8569	.3961	.9022	2.307	.4236
3/4	.7500	.5625	.8660	.4219	.9086	2.356	.4418
49/64	.765625	.5862	.8750	.4488	.9148	2.405	.4604
25/32	.78125	.6104	.8839	.4768	.9210	2.454	.4794
51/64	.796875	.6350	.8927	.5060	.9271	2.503	.4987
13/16	.8125	.6602	.9014	.5364	.9331	2.553	.5185
53/64	.828125	.6858	.9100	.5679	.9391	2.602	.5386
27/32	.84375	.7119	.9186	.6007	.9449	2.651	.5592
55/64	.859375	.7385	.9270	.6347	.9507	2.700	.5801
7/8	.8750	.7656	.9354	.6699	.9565	2.749	.6013
57/64	.890625	.7932	.9437	.7064	.9621	2.798	.6230
29/32	.90625	.8213	.9520	.7443	.9677	2.847	.6450
59/64	.921875	.8499	.9601	.7835	.9732	2.896	.6675
15/16	.9375	.8789	.9682	.8240	.9787	2.945	.6903
61/64	.953125	.9084	.9763	.8659	.9841	2.994	.7135
31/32	.96875	.9385	.9843	.9091	.9895	3.043	.7371
63/64	.984375	.9690	.9922	.9539	.9948	3.093	.7610
1	1	1	1	1	1	3.1416	.7854

TABLE II

SQUARES, CUBES, SQUARE AND CUBE ROOTS OF NUMBERS FROM 1 TO 500

No.	Square	Cube	Sq. Root	Cube Root	No.	Square	Cube	Sq. Root	Cube Root
1	1	1	1.0000	1.0000	51	2601	132651	7.1414	3.7084
2	4	8	1.4142	1.2599	52	2704	140608	7.2111	3.7325
3	9	27	1.7321	1.4422	53	2809	148877	7.2801	3.7563
4	16	64	2.0000	1.5874	54	2916	157464	7.3485	3.7798
5	25	125	2.2361	1.7100	55	3025	166375	7.4162	3.8030
6	36	216	2.4495	1.8171	56	3136	175616	7.4833	3.8259
7	49	343	2.6458	1.9129	57	3249	185193	7.5498	3.8485
8	64	512	2.8284	2.0000	58	3364	195112	7.6158	3.8709
9	81	729	3.0000	2.0801	59	3481	205379	7.6811	3.8930
10	100	1000	3.1623	2.1544	60	3600	216000	7.7460	3.9149
11	121	1331	3.3166	2.2240	61	3721	226981	7.8102	3.9365
12	144	1728	3.4641	2.2894	62	3844	238328	7.8740	3.9579
13	169	2197	3.6056	2.3513	63	3969	250047	7.9373	3.9791
14	196	2744	3.7417	2.4101	64	4096	262144	8.0000	4.0000
15	225	3375	3.8730	2.4662	65	4225	274625	8.0623	4.0207
16	256	4096	4.0000	2.5198	66	4356	287496	8.1240	4.0412
17	289	4913	4.1231	2.5713	67	4489	300763	8.1854	4.0615
18	324	5832	4.2426	2.6207	68	4624	314432	8.2462	4.0817
19	361	6859	4.3589	2.6684	69	4761	328509	8.3066	4.1016
20	400	8000	4.4721	2.7144	70	4900	343000	8.3666	4.1213
21	441	9261	4.5826	2.7589	71	5041	357911	8.4261	4.1408
22	484	10648	4.6904	2.8020	72	5184	373248	8.4853	4.1602
23	529	12167	4.7958	2.8439	73	5329	389017	8.5440	4.1793
24	576	13824	4.8990	2.8845	74	5476	405224	8.6023	4.1983
25	625	15625	5.0000	2.9240	75	5625	421875	8.6603	4.2172
26	676	17576	5.0990	2.9625	76	5776	438976	8.7178	4.2358
27	729	19683	5.1962	3.0000	77	5929	456533	8.7750	4.2543
28	784	21952	5.2915	3.0366	78	6084	474552	8.8318	4.2727
29	841	24389	5.3852	3.0723	79	6241	493039	8.8882	4.2908
30	900	27000	5.4772	3.1072	80	6400	512000	8.9443	4.3089
31	961	29791	5.5678	3.1414	81	6561	531441	9.0000	4.3267
32	1024	32768	5.6569	3.1748	82	6724	551368	9.0554	4.3445
33	1089	35937	5.7446	3.2075	83	6889	571787	9.1104	4.3621
34	1156	39304	5.8310	3.2396	84	7056	592704	9.1652	4.3795
35	1225	42875	5.9161	3.2711	85	7225	614125	9.2195	4.3968
36	1296	46656	6.0000	3.3019	86	7396	636056	9.2736	4.4140
37	1369	50653	6.0828	3.3322	87	7569	658503	9.3276	4.4310
38	1444	54872	6.1644	3.3620	88	7744	681472	9.3808	4.4480
39	1521	59319	6.2450	3.3912	89	7921	704969	9.4340	4.4647
40	1600	64000	6.3246	3.4200	90	8100	729000	9.4868	4.4814
41	1681	68921	6.4031	3.4482	91	8281	753571	9.5394	4.4979
42	1764	74088	6.4807	3.4760	92	8464	778688	9.5917	4.5144
43	1849	79507	6.5574	3.5034	93	8649	804357	9.6437	4.5307
44	1936	85184	6.6332	3.5303	94	8836	830584	9.6954	4.5468
45	2025	91125	6.7082	3.5569	95	9025	857375	9.7468	4.5629
46	2116	97336	6.7823	3.5830	96	9216	884736	9.7980	4.5789
47	2209	103823	6.8557	3.6088	97	9409	912673	9.8489	4.5947
48	2304	110592	6.9282	3.6342	98	9604	941192	9.8995	4.6104
49	2401	117649	7.0000	3.6593	99	9801	970299	9.9499	4.6261
50	2500	125000	7.0711	3.6840	100	10000	1000000	10.0000	4.6416

TABLE II—*Continued*

SQUARES, CUBES, SQUARE AND CUBE ROOTS OF NUMBERS FROM 1 TO 500

No.	Square	Cube	Sq. Root	Cube Root	No.	Square	Cube	Sq. Root	Cube Root
101	10201	1030301	10.0499	4.6570	151	22801	3442951	12.2882	5.3251
102	10404	1061208	10.0995	4.6723	152	23104	3511808	12.3288	5.3368
103	10609	1092727	10.1489	4.6875	153	23409	3581577	12.3693	5.3485
104	10816	1124864	10.1980	4.7027	154	23716	3652264	12.4097	5.3601
105	11025	1157625	10.2470	4.7177	155	24025	3723875	12.4499	5.3717
106	11236	1191016	10.2956	4.7326	156	24336	3796416	12.4900	5.3832
107	11449	1225043	10.3441	4.7475	157	24649	3869893	12.5300	5.3947
108	11664	1259712	10.3923	4.7622	158	24964	3944312	12.5698	5.4061
109	11881	1295029	10.4403	4.7769	159	25281	4019679	12.6095	5.4175
110	12100	1331000	10.4881	4.7914	160	25600	4096000	12.6491	5.4288
111	12321	1367631	10.5357	4.8059	161	25921	4173281	12.6886	5.4401
112	12544	1404928	10.5830	4.8203	162	26244	4251528	12.7279	5.4514
113	12769	1442897	10.6301	4.8346	163	26569	4330747	12.7671	5.4626
114	12996	1481544	10.6771	4.8488	164	26896	4410944	12.8062	5.4737
115	13225	1520875	10.7238	4.8629	165	27225	4492125	12.8452	5.4848
116	13456	1560896	10.7703	4.8770	166	27556	4574296	12.8841	5.4959
117	13689	1601613	10.8167	4.8910	167	27889	4657463	12.9228	5.5069
118	13924	1643032	10.8628	4.9049	168	28224	4741632	12.9615	5.5178
119	14161	1685159	10.9087	4.9187	169	28561	4826809	13.0000	5.5288
120	14400	1728000	10.9545	4.9324	170	28900	4913000	13.0384	5.5397
121	14641	1771561	11.0000	4.9461	171	29241	5000211	13.0767	5.5505
122	14884	1815848	11.0454	4.9597	172	29584	5088448	13.1149	5.5613
123	15129	1860867	11.0905	4.9732	173	29929	5177717	13.1529	5.5721
124	15376	1906624	11.1355	4.9866	174	30276	5268024	13.1909	5.5828
125	15625	1953125	11.1803	5.0000	175	30625	5359375	13.2288	5.5934
126	15876	2000376	11.2250	5.0133	176	30976	5451776	13.2665	5.6041
127	16129	2048383	11.2694	5.0265	177	31329	5545233	13.3041	5.6147
128	16384	2097152	11.3137	5.0397	178	31684	5639752	13.3417	5.6252
129	16641	2146689	11.3578	5.0528	179	32041	5735339	13.3791	5.6357
130	16900	2197000	11.4018	5.0658	180	32400	5832000	13.4164	5.6462
131	17161	2248091	11.4455	5.0788	181	32761	5929741	13.4536	5.6567
132	17424	2299968	11.4891	5.0916	182	33124	6028568	13.4907	5.6671
133	17689	2352637	11.5326	5.1045	183	33489	6128487	13.5277	5.6774
134	17956	2406104	11.5758	5.1172	184	33856	6229504	13.5647	5.6877
135	18225	2460375	11.6190	5.1299	185	34225	6331625	13.6015	5.6980
136	18496	2515456	11.6619	5.1426	186	34596	6434856	13.6382	5.7083
137	18769	2571353	11.7047	5.1551	187	34969	6539203	13.6748	5.7185
138	19044	2628072	11.7473	5.1676	188	35344	6644672	13.7113	5.7287
139	19321	2685619	11.7898	5.1801	189	35721	6751269	13.7477	5.7388
140	19600	2744000	11.8322	5.1925	190	36100	6859000	13.7840	5.7489
141	19881	2803221	11.8743	5.2048	191	36481	6967871	13.8203	5.7590
142	20164	2863288	11.9164	5.2171	192	36864	7077888	13.8564	5.7690
143	20449	2924207	11.9583	5.2293	193	37249	7189057	13.8924	5.7790
144	20736	2985984	12.0000	5.2415	194	37636	7301384	13.9284	5.7890
145	21025	3048625	12.0416	5.2536	195	38025	7414875	13.9642	5.7089
146	21316	3112136	12.0830	5.2656	196	38416	7529536	14.0000	5.8088
147	21609	3176523	12.1244	5.2776	197	38809	7645373	14.0357	5.8186
148	21904	3241792	12.1655	5.2896	198	39204	7762392	14.0712	5.8285
149	22201	3307949	12.2066	5.3015	199	39601	7880599	14.1067	5.8383
150	22500	3375000	12.2474	5.3133	200	40000	8000000	14.1421	5.8480

TABLE II—*Continued*

SQUARES, CUBES, SQUARE AND CUBE ROOTS OF NUMBERS FROM 1 TO 500

No.	Square	Cube	Sq. Root	Cube Root	No.	Square	Cube	Sq. Root	Cube Root
201	40401	8120601	14.1774	5.8578	251	63001	15813251	15.8430	6.3080
202	40804	8242408	14.2127	5.8675	252	63504	16003008	15.8745	6.3164
203	41209	8365427	14.2478	5.8771	253	64009	16194277	15.9060	6.3247
204	41616	8489664	14.2829	5.8868	254	64516	16387064	15.9374	6.3330
205	42025	8615125	14.3178	5.8964	255	65025	16581375	15.9687	6.3413
206	42436	8741816	14.3527	5.9059	256	65536	16777216	16.0000	6.3496
207	42849	8869743	14.3875	5.9155	257	66049	16974593	16.0312	6.3579
208	43264	8998912	14.4222	5.9250	258	66564	17173512	16.0624	6.3661
209	43681	9129329	14.4568	5.9345	259	67081	17373979	16.0935	6.3743
210	44100	9261000	14.4914	5.9439	260	67600	17576000	16.1245	6.3825
211	44521	9393931	14.5258	5.9533	261	68121	17779581	16.1555	6.3907
212	44944	9528128	14.5602	5.9627	262	68644	17984728	16.1864	6.3988
213	45369	9663597	14.5945	5.9721	263	69169	18191447	16.2173	6.4070
214	45796	9800344	14.6287	5.9814	264	69696	18399744	16.2481	6.4151
215	46225	9938375	14.6629	5.9907	265	70225	18609625	16.2788	6.4232
216	46656	10077696	14.6969	6.0000	266	70756	18821096	16.3095	6.4312
217	47089	10218313	14.7309	6.0092	267	71289	19034163	16.3401	6.4393
218	47524	10360232	14.7648	6.0185	268	71824	19248832	16.3707	6.4473
219	47961	10503459	14.7986	6.0277	269	72361	19465109	16.4012	6.4553
220	48400	10648000	14.8324	6.0368	270	72900	19683000	16.4317	6.4633
221	48841	10793861	14.8661	6.0459	271	73441	19902511	16.4621	6.4713
222	49284	10941048	14.8997	6.0550	272	73984	20123648	16.4924	6.4792
223	49729	11089567	14.9332	6.0641	273	74529	20346417	16.5227	6.4872
224	50176	11239424	14.9666	6.0732	274	75076	20570824	16.5529	6.4951
225	50625	11390625	15.0000	6.0822	275	75625	20796875	16.5831	6.5030
226	51076	11543176	15.0333	6.0912	276	76176	21024576	16.6132	6.5108
227	51529	11697083	15.0665	6.1002	277	76729	21253933	16.6433	6.5187
228	51984	11852352	15.0997	6.1091	278	77284	21484952	16.6733	6.5265
229	52441	12008989	15.1327	6.1180	279	77841	21717639	16.7033	6.5343
230	52900	12167000	15.1658	6.1269	280	78400	21952000	16.7332	6.5421
231	53361	12326391	15.1987	6.1358	281	78961	22188041	16.7631	6.5499
232	53824	12487168	15.2315	6.1446	282	79524	22425768	16.7929	6.5577
233	54289	12649337	15.2643	6.1534	283	80089	22665187	16.8226	6.5654
234	54756	12812904	15.2971	6.1622	284	80656	22906304	16.8523	6.5731
235	55225	12977875	15.3297	6.1710	285	81225	23149125	16.8819	6.5808
236	55696	13144256	15.3623	6.1797	286	81796	23393656	16.9115	6.5885
237	56169	13312053	15.3948	6.1885	287	82369	23639903	16.9411	6.5962
238	56644	13481272	15.4272	6.1972	288	82944	23887872	16.9706	6.6039
239	57121	13651919	15.4596	6.2058	289	83521	24137569	17.0000	6.6115
240	57600	13824000	15.4919	6.2145	290	84100	24389000	17.0294	6.6191
241	58081	13997521	15.5242	6.2231	291	84681	24642171	17.0587	6.6267
242	58564	14172488	15.5563	6.2317	292	85264	24897088	17.0880	6.6343
243	59049	14348907	15.5885	6.2403	293	85849	25153757	17.1172	6.6419
244	59536	14526784	15.6205	6.2488	294	86436	25412184	17.1464	6.6494
245	60025	14706125	15.6525	6.2573	295	87025	25672375	17.1756	6.6569
246	60516	14886936	15.6844	6.2658	296	87616	25934336	17.2047	6.6644
247	61009	15069223	15.7162	6.2743	297	88209	26198073	17.2337	6.6719
248	61504	15252992	15.7480	6.2828	298	88804	26463592	17.2627	6.6794
249	62001	15438249	15.7797	6.2912	299	89401	26730899	17.2916	6.6869
250	62500	15625000	15.8114	6.2996	300	90000	27000000	17.3205	6.6943

TABLE II—*Continued*

SQUARES, CUBES, SQUARE AND CUBE ROOTS OF NUMBERS FROM 1 TO 500

No.	Square	Cube	Sq. Root	Cube Root	No.	Square	Cube	Sq. Root	Cube Root
301	90601	27270901	17.3494	6.7018	351	123201	43243551	18.7350	7.0540
302	91204	27543608	17.3781	6.7092	352	123904	43614208	18.7617	7.0607
303	91809	27818127	17.4069	6.7166	353	124609	43986977	18.7883	7.0674
304	92416	28094464	17.4356	6.7240	354	125316	44361864	18.8149	7.0740
305	93025	28372625	17.4642	6.7313	355	126025	44738875	18.8414	7.0807
306	93636	28652616	17.4929	6.7387	356	126736	45118016	18.8680	7.0873
307	94249	28934443	17.5214	6.7460	357	127449	45499293	18.8944	7.0940
308	94864	29218112	17.5499	6.7533	358	128164	45882712	18.9209	7.1006
309	95481	29503629	17.5784	6.7606	359	128881	46268279	18.9473	7.1072
310	96100	29791000	17.6068	6.7679	360	129600	46656000	18.9737	7.1138
311	96721	30080231	17.6352	6.7752	361	130321	47045881	19.0000	7.1204
312	97344	30371328	17.6635	6.7824	362	131044	47437928	19.0263	7.1269
313	97969	30664297	17.6918	6.7897	363	131769	47832147	19.0526	7.1335
314	98596	30959144	17.7200	6.7969	364	132496	48228544	19.0788	7.1400
315	99225	31255875	17.7482	6.8041	365	133225	48627125	19.1050	7.1466
316	99856	31554496	17.7764	6.8113	366	133956	49027896	19.1311	7.1531
317	100489	31855013	17.8045	6.8185	367	134689	49430863	19.1572	7.1596
318	101124	32157432	17.8326	6.8256	368	135424	49836032	19.1833	7.1661
319	101761	32461759	17.8606	6.8328	369	136161	50243409	19.2094	7.1726
320	102400	32768000	17.8885	6.8399	370	136900	50653000	19.2354	7.1791
321	103041	33076161	17.9165	6.8470	371	137641	51064811	19.2614	7.1855
322	103684	33386248	17.9444	6.8541	372	138384	51478848	19.2873	7.1920
323	104329	33698267	17.9722	6.8612	373	139129	51895117	19.3132	7.1984
324	104976	34012224	18.0000	6.8683	374	139876	52313624	19.3391	7.2048
325	105625	34328125	18.0278	6.8753	375	140625	52734375	19.3649	7.2112
326	106276	34645976	18.0555	6.8824	376	141376	53157376	19.3907	7.2177
327	106929	34965783	18.0831	6.8894	377	142129	53582633	19.4165	7.2240
328	107584	35287552	18.1108	6.8964	378	142884	54010152	19.4422	7.2304
329	108241	35611289	18.1384	6.9034	379	143641	54439939	19.4679	7.2368
330	108900	35937000	18.1659	6.9104	380	144400	54872000	19.4936	7.2432
331	109561	36264691	18.1934	6.9174	381	145161	55306341	19.5192	7.2495
332	110224	36594368	18.2209	6.9244	382	145924	55742968	19.5448	7.2558
333	110889	36926037	18.2483	6.9313	383	146689	56181887	19.5704	7.2622
334	111556	37259704	18.2757	6.9382	384	147456	56623104	19.5959	7.2685
335	112225	37595375	18.3030	6.9451	385	148225	57066625	19.6214	7.2748
336	112896	37933056	18.3303	6.9521	386	148996	57512456	19.6469	7.2811
337	113569	38272753	18.3576	6.9589	387	149769	57960603	19.6723	7.2874
338	114244	38614472	18.3848	6.9658	388	150544	58411072	19.6977	7.2936
339	114921	38958219	18.4120	6.9727	389	151321	58863869	19.7231	7.2999
340	115600	39304000	18.4391	6.9795	390	152100	59319000	19.7484	7.3061
341	116281	39651821	18.4662	6.9864	391	152881	59776471	19.7737	7 3124
342	116964	40001688	18.4932	6.9932	392	153664	60236288	19.7990	7.3186
343	117649	40353607	18.5203	7.0000	393	154449	60698457	19.8242	7.3248
344	118336	40707584	18.5472	7.0068	394	155236	61162984	19.8494	7.3310
345	119025	41063625	18.5742	7.0136	395	156025	61629875	19.8746	7.3372
346	119716	41421736	18.6011	7.0203	396	156816	62099136	19.8997	7.3434
347	120409	41781923	18.6279	7.0271	397	157609	62570773	19.9249	7.3496
348	121104	42144192	18.6548	7.0338	398	158404	63044792	19.9499	7.3558
349	121801	42508549	18.6815	7.0406	399	159201	63521199	19.9750	7.3619
350	122500	42875000	18.7083	7.0473	400	160000	64000000	20.0000	7.3681

TABLE II—*Concluded*

SQUARES, CUBES, SQUARE AND CUBE ROOTS OF NUMBERS FROM 1 TO 500

No.	Square	Cube	Sq. Root	Cube Root	No.	Square	Cube	Sq. Root	Cube Root
401	160801	64481201	20.0250	7.3742	451	203401	91733851	21.2368	7.6688
402	161604	64964808	20.0499	7.3803	452	204304	92345408	21.2603	7.6744
403	162409	65450827	20.0749	7.3864	453	205209	92959677	21.2838	7.6800
404	163216	65939264	20.0998	7.3925	454	206116	93576664	21.3073	7.6857
405	164025	66430125	20.1246	7.3986	455	207025	94196375	21.3307	7.6914
406	164836	66923416	20.1494	7.4047	456	207936	94818816	21.3542	7.6970
407	165649	67419143	20.1742	7.4108	457	208849	95443993	21.3776	7.7026
408	166464	67917312	20.1990	7.4169	458	209764	96071912	21.4009	7.7082
409	167281	68417929	20.2237	7.4229	459	210681	96702579	21.4243	7.7138
410	168100	68921000	20.2485	7.4290	460	211600	97336000	21.4476	7.7194
411	168921	69426531	20.2731	7.4350	461	212521	97972181	21.4709	7.7250
412	169744	69934528	20.2978	7.4410	462	213444	98611128	21.4942	7.7306
413	170569	70444997	20.3224	7.4470	463	214369	99252847	21.5174	7.7362
414	171396	70957944	20.3470	7.4530	464	215296	99897344	21.5407	7.7418
415	172225	71473375	20.3715	7.4590	465	216225	100544625	21.5639	7.7473
416	173056	71991296	20.3961	7.4650	466	217156	101194696	21.5870	7.7529
417	173889	72511713	20.4206	7.4710	467	218089	101847563	21.6102	7.7584
418	174724	73034632	20.4450	7.4770	468	219024	102503232	21.6333	7.7639
419	175561	73560059	20.4695	7.4829	469	219961	103161709	21.6564	7.7695
420	176400	74088000	20.4939	7.4889	470	220900	103823000	21.6795	7.7750
421	177241	74618461	20.5183	7.4948	471	221841	104487111	21.7025	7.7805
422	178084	75151448	20.5426	7.5007	472	222784	105154048	21.7256	7.7860
423	178929	75686967	20.5670	7.5067	473	223729	105823817	21.7486	7.7915
424	179776	76225024	20.5913	7.5126	474	224676	106496424	21.7715	7.7970
425	180625	76765625	20.6155	7.5185	475	225625	107171875	21.7945	7.8025
426	181476	77308776	20.6398	7.5244	476	226576	107850176	21.8174	7.8079
427	182329	77854483	20.6640	7.5302	477	227529	108531333	21.8403	7.8134
428	183184	78402752	20.6882	7.5361	478	228484	109215352	21.8632	7.8188
429	184041	78953589	20.7123	7.5420	479	229441	109902239	21.8861	7.8243
430	184900	79507000	20.7364	7.5478	480	230400	110592000	21.9089	7.8297
431	185761	80062991	20.7605	7.5537	481	231361	111284641	21.9317	7.8352
432	186624	80621568	20.7846	7.5595	482	232324	111980168	21.9545	7.8406
433	187489	81182737	20.8087	7.5654	483	233289	112678587	21.9773	7.8460
434	188356	81746504	20.8327	7.5712	484	234256	113379904	22.0000	7.8514
435	189225	82312875	20.8567	7.5770	485	235225	114084125	22.0227	7.8568
436	190096	82881856	20.8806	7.5828	486	236196	114791256	22.0454	7.8622
437	190969	83453453	20.9045	7.5886	487	237169	115501303	22.0681	7.8676
438	191844	84027672	20.9284	7.5944	488	238144	116214272	22.0907	7.8730
439	192721	84604519	20.9523	7.6001	489	239121	116930169	22.1133	7.8784
440	193600	85184000	20.9762	7.6059	490	240100	117649000	22.1359	7.8837
441	194481	85766121	21.0000	7.6117	491	241081	118370771	22.1585	7.8891
442	195364	86350888	21.0238	7.6174	492	242064	119095488	22.1811	7.8944
443	196249	86938307	21.0476	7.6232	493	243049	119823157	22.2036	7.8998
444	197136	87528384	21.0713	7.6289	494	244036	120553784	22.2261	7.9051
445	198025	88121125	21.0950	7.6346	495	245025	121287375	22.2486	7.9105
446	198916	88716536	21.1187	7.6403	496	246016	122023936	22.2711	7.9158
447	199809	89314623	21.1424	7.6460	497	247009	122763473	22.2935	7.9211
448	200704	89915392	21.1660	7.6517	498	248004	123505992	22.3159	7.9264
449	201601	90518849	21.1896	7.6574	499	249001	124251499	22.3383	7.9317
450	202500	91125000	21.2132	7.6631	500	250000	125000000	22.3607	7.9370

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